ELECTROPHYSICS – Ph. D. Qualifying Exam Spring 2008

[See page 2 for table of constants, relationships and vector operators.]

1. Two co-axial cylindrical conductors are separated by a lossy dielectric. The inner conductor has a radius r = a = 0.5 cm (o.d. = 1 cm). The outer conductor has a radius r = b = 2 cm (i.d. = 4 cm). A potential of $\Phi = V_o$ is applied to the inner conductor. The outer conductor is grounded ($\Phi = 0$). There is no variation with time, with the axial co-ordinate or with the azimuthal co-ordinate.

   (i) Write the equation that can be used to determine the radial dependence of the potential Φ in the dielectric.
   (2 points)

   (ii) Solve the equation specified in part (i) to obtain an explicit equation for $\Phi(r)$.
   (3 points)

   (iii) Apply the boundary conditions at r = a and r = b to evaluate the constants of integration
   (3 points)

   (iv) Derive an expression the electric field in the dielectric as a function of r and determine its numerical value at r = a if V_o = 10 Volts.
   (3 points)

2. A plane wave is normally incident from vacuum onto a planar conducting surface. The wave frequency is f = 10 GHz.

   (i) If the power density in the incoming wave is 1 megawatt per square meter, calculate the magnitude of the magnetic field in the incoming wave and the magnitude of the surface current density (assuming the conductor is perfect.
   (3 points)

   (ii) If the conductor is copper (conductivity $\sigma = 6 \times 10^7$ S/m), what is the characteristic depth of penetration of the surface current into the conductor and what is the surface resistance?
   (3 points)

   (iii) Estimate the power loss per unit area deposited by the wave near the surface of the copper and the power density in the reflected wave
   (3 points)

I. CONSTANTS AND RELATIONSHIPS

Permittivity of free space $\varepsilon_o = (36\,\pi \times 10^9)^{-1}$ Farads / meter
Permeability of free space $\mu_o = 4\,\pi \times 10^{-7}$ Henries / meter
Wave impedance of free space $Z_o = (\mu_o / \varepsilon_o)^{1/2} = 120\,\pi$ Ohms
Skin depth $\delta_s = (\pi\, f\, \mu_o\, \sigma)^{-1/2}$

II. VECTOR OPERATORS in

CYLINDRICAL COORDINATES

Coordinates (r, φ, z) Unit vectors $(\vec{i}_1, \vec{i}_2, \vec{i}_3)$

Gradient

$$\nabla f = \vec{i}_1 \frac{\partial f}{\partial r} + \vec{i}_2 \frac{1}{r}\frac{\partial f}{\partial \varphi} + \vec{i}_3 \frac{\partial f}{\partial z}$$

Curl

$$\nabla \times \vec{A} = \vec{i}_1 \left(\frac{1}{r}\frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z}\right) + \vec{i}_2 \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \vec{i}_3 \left(\frac{1}{r}\frac{\partial}{\partial r}(rA_\varphi) - \frac{1}{r}\frac{\partial A_r}{\partial \varphi}\right)$$

Divergence

$$\nabla \cdot \vec{A} = \frac{1}{r}\frac{\partial}{\partial r}(r\,A_r) + \frac{1}{r}\frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z}$$

Laplacian

$$\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \varphi^2} + \frac{\partial^2 f}{\partial z^2}$$