

**PROBABILITY—Ph.D. Qualifying Exam Spring 2007**

A postman delivers mail daily starting on Day 1. The conditional probability that he is bitten by a dog on Day  $n$ , given that he has not been bitten on any of the preceding days, equals  $q_n$ . Let  $X$  be the number of days until the postman sustains his first bite (i.e., the first bite occurs on Day  $X$ ).

(i) (5 pts.) Write an expression for  $P(X > n)$  in terms of  $q_i$ 's.

(ii) (5 pts.) Write an expression for  $P(X = n)$  in terms of  $q_i$ 's.

For the remainder of the problem, let  $q_n = \alpha$  for all  $n$ , where  $0 < \alpha < 1$ .

(iii) (4 pts.) What is the probability that the postman is *never* bitten by a dog? Justify your answer.

(iv) (6 pts.) Determine the expected number of days  $E[X]$  until the first bite.