

PROBABILITY — Ph.D. Qualifying Exam Fall 2007**Part A**

Consider a random byte $\mathbf{X} = (X_1, \dots, X_8)$ in which the bits X_i are independent and such that $\Pr(X_i = 0) = 0.4$ and $\Pr(X_i = 1) = 0.6$ for all i . Due to a fault in the memory, any single 1 may be erroneously recorded as a 0 with probability $1/4$, and such errors occur independently from bit to bit; 0's are always correctly recorded. Denote the recorded byte by $\mathbf{Y} = (Y_1, \dots, Y_8)$.

(A1) (3 pts.) Given that $\mathbf{X} = 11111111$, compute the conditional probability that \mathbf{Y} consists of exactly six 1's and two 0's (regardless of ordering).

(A2) (7 pts.) Given that \mathbf{Y} consists of exactly six 1's and two 0's (regardless of ordering), compute the conditional probability that $\mathbf{X} = 11111111$.

Part B

Suppose that there are only two manufacturers of generic hard drives, M_1 and M_2 , supplying $3/5$ and $2/5$ (respectively) of the drives available in the market. The lifetimes of these two types of drives are random, with probability density functions

$$f_1(t) = e^{-t} \quad \text{and} \quad f_2(t) = 2e^{-2t}$$

respectively, where $t \geq 0$ is in decades (1 decade = 10 years). The two types of drives are identical in appearance and weight.

(B1) (5 pts.) What is the probability that a randomly purchased hard drive will last five years or longer?

(B2) (5 pts.) Given that a hard drive lasted exactly six years, what is the probability that it was manufactured by M_1 ?