LINEAR SYSTEMS—Ph.D. Qualifying Exam Fall 2007

Part A (3 pts.)

A discrete-time system has impulse response

$$h[n] = a^n u[n+2], \quad |a| < 1$$

Is this system bounded-input, bounded-output stable? Is it causal? Is it memoryless?

Part B (5 pts.)

Consider a system H described by the differential equation

$$\frac{dy(t)}{dt} + 2y(t) = \frac{d^2x(t)}{dt^2} + 2\frac{dx(t)}{dt} - 3x(t)$$

Find the transfer function $\tilde{H}(s)$ of an inverse system for H. Does a stable and causal inverse system exist?

Part C (8 pts.)

A discrete-time system has system (transfer) function

$$H(z) = \frac{2}{1 - 0.9e^{j(\pi/4)}z^{-1}} + \frac{2}{1 - 0.9e^{-j(\pi/4)}z^{-1}} + \frac{3}{1 + 2z^{-1}}$$

Find the impulse response $h[n]$ assuming the system is (i) stable and (ii) causal. Can this system be both stable and causal?

Part D (4 pts.)

Determine the sequence $x[n]$ whose discrete-time Fourier transform over the frequency interval $[-\pi, \pi]$ is given by

$$X(e^{j\omega}) = \begin{cases} 1, & |\omega| \le \alpha; \\ 0, & \alpha < |\omega| \le \pi \end{cases}$$

(where $0 < \alpha < \pi$). Hence, or otherwise, evaluate

$$E = \sum_{n=-\infty}^{\infty} \frac{\sin^2(\alpha n)}{\pi^2 n^2}$$