

LINEAR SYSTEMS—Ph.D. Qualifying Exam Fall 2006

A stable (in the BIBO sense) linear time-invariant system operating in discrete time has impulse response $h[n]$ and frequency response given by

$$H(e^{j\omega}) = \frac{4}{5 - 3 \cos \omega}$$

where ω is in radians (per sample).

(i) (3 pts.) Sketch $H(e^{j\omega})$ for $\omega \in (-\pi, \pi]$. Based on the form of $H(e^{j\omega})$, what can you say about the form of the impulse response $h[n]$? Is the system causal?

(ii) (5 pts.) Derive the system function $H(z)$ and determine its poles, its zeros and its region of convergence. (*Check: Your answers should have rational values.*)

(iii) (6 pts.) Determine the impulse response $h[n]$.

(iv) (6 pts.) Find three input signals $x_1[n]$, $x_2[n]$ and $x_3[n]$ such that the corresponding system outputs are:

$$y_1[n] = 3^{-n} \cdot u[n] , \quad y_2[n] = 3^n \cdot u[-n] \quad \text{and} \quad y_3[n] = \delta[n]$$

(As usual, $\delta[n]$ is a unit impulse, and $u[n]$ a unit step, at the origin.)

$$\left(H(z) = \sum_{n=-\infty}^{\infty} h[n]z^{-n} , \quad \sum_{k=0}^{\infty} \alpha^k = \frac{1}{1 - \alpha} \quad \text{for} \quad |\alpha| < 1 \right)$$