**DIGITAL LOGIC — Ph.D. Qualifying Exam Spring 2006**

1. (8 points) Design a circuit that tells the number of days in a given month. For example, when input is 1 (for January), your circuit should output 31. You can assume that February always has 28 days.

   a. (4 points) Draw the truth table for this circuit, assuming both input and output are in binary.

   b. (4 points) Simplify the output to the format of **sum of product** with minimum number of literals. (You do not have to draw the circuit.)

2. (4 points) A gate or a set of gates is called universal if it can implement all digital system (Boolean function). A standard way to show that a gate or a set of gates is universal is to show that it can implement the standard universal gates {AND, OR, NOT}.

   A multiplexer is a circuit that outputs one of many inputs based on selection control. A 2-to-1 multiplexer has two inputs, x and y, and one selection bit, s. Its output z will be x if s=0 and y if s=1.

   a. (1 point) Give a Boolean expression of the above 2-to-1 multiplexer.

   b. (3 point) Show that the 2-to-1 multiplexer is universal.

3. (8 points) Analyze the following sequential circuit by deriving the output, flip-flop input and next state functions in the **sum of product** form. You don't need to draw the state transition graph/table.

   x and y are the inputs, $z_1$ and $z_2$ are the outputs. T flip-flop's next state is given by: $Q(t+1) = T \oplus Q(t)$, where $\oplus$ is the logic XOR operator defined as $x \oplus y = x'y + xy'$