

Problem 1 (10 pts)

Suppose \mathbf{A} is 2×2 matrix that has the following properties:

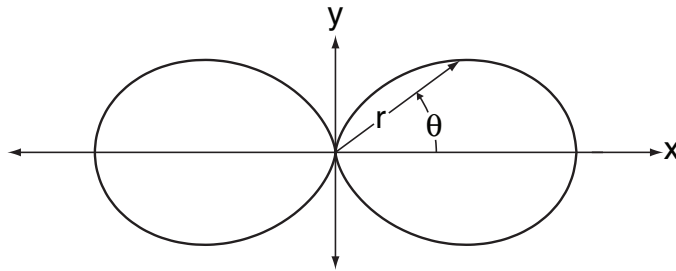
- \mathbf{A} is real and symmetric
- The vector $\mathbf{v}_1 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ is an eigenvector of \mathbf{A} with eigenvalue of $\lambda_1 = 5$
- The determinant of \mathbf{A} is 50

Based on the information given above, determine the matrix \mathbf{A} .

Problem 2 (10 pts)

Consider the figure-eight curve in the x - y plane described by the parametric equation:

$$r(\theta) = a \cos^2 \theta, \quad 0 \leq \theta < 2\pi$$



Determine the total path length of this curve.

Hint: You may find the following indefinite integral helpful:

$$\int \sqrt{u^2 \pm \alpha^2} du = \frac{u}{2} \sqrt{u^2 \pm \alpha^2} \pm \frac{\alpha^2}{2} \ln \left| u + \sqrt{u^2 \pm \alpha^2} \right|$$