Homework 1  (partial; more will be assigned on February 1)

1. The robot model \((\ast)\) implies

\[
\dot{x}_1 \sin(x_3) - \dot{x}_2 \cos(x_3) = 0
\]

Does this mean that the system is nonholonomic?

2. The set of matrices of the form

\[
g = \begin{bmatrix}
\cos(x_3) & -\sin(x_3) \\
\sin(x_3) & \cos(x_3)
\end{bmatrix}
\]

is a group in the sense that it is closed under multiplication, each element of the set has an inverse in the set, and there is a unique identity element. Verify this.

3. We denote the set in problem 2 as \(SE(2)\) (for special Euclidean matrix Lie group of rigid motions in the plane), show that the robot model is equivalent to
\[ \hat{\mathbf{g}} = g \left( u \mathbf{X}_1 + v \mathbf{X}_2 \right) \]

For suitable 3x3 matrices \( \mathbf{X}_1 \) and \( \mathbf{X}_2 \).

Determine \( \mathbf{X}_1 \) and \( \mathbf{X}_2 \).