Problem 1

For the time-invariant system

\[ x = Ax + Bu + cv \]

with a known disturbance \( V(t) \), suggest / derive a control that drives the system from \((x_0, 0)\) to \((0, T)\) and minimizes

\[ \eta = \int_{0}^{T} u'(t) u(t) \, dt \]

state any needed hypotheses clearly.

Problem 2

Complete the proof of the necessary conditions theorem in Lecture Notes 3 (page 5, part (c)).

Problem 3

Consider the problem of finding an optimal control for the system:

\[ \dot{x} = -\alpha x + u \quad x > 0; \]

\[ \eta = \int_{0}^{1} (x^2 + u^2) \, dt + x^2(1); \]

with initial \( x(0) = 1 \).

Use a numerical approach based on canonical equation (and MATLAB) to investigate this problem:

what is the optimal value \( \eta_{\text{min}} \)?