ENEE 664 Homework Set 4 due: 02/23/2012.

1. Consider the problem of finding a time function $x(t)$ such that $x(0) = 1$ and $x(2\pi) = 0$ and the functional

$$\eta = \int_0^{2\pi} (\dot{x}^2 - 2\cos(t)\,\dot{x})\,dt$$

is to be minimized. Use optimal control theory to solve this problem.

2. Consider the scalar system $\dot{x} = u$. It is desired to find a feedback control law for $u(\cdot)$ such that the state x is regulated around a constant level ℓ. Determine such a feedback law by minimizing

$$\int_0^T (u^2(t) + (x(t)-\ell)^2)\,dt.$$

Does the choice of time horizon matter?

3. Consider the optimal control problem

$$\text{Min } J[u] = \int_{t_0}^{t_1} \left[x^T(t)\, L(t)\, x(t) + u^T(t)\, u(t)\right] dt$$

$$+ \left(x(t_1) - d\right)^T Q \left(x(t_1) - d\right)$$

Subject to

$$\dot{x}(t) = A(t)\, x(t) + B(t)\, u(t)$$

$$x(t_0) = x_0$$

Here x_0, $L = L^T$, A, B, Q and d are given.

Show that the optimal control has the form

$$u(t) = -B^T(t)\left(K(t)\, x(t) + \tfrac{1}{2}\eta(t)\right)$$

where K and η satisfy suitable hypotheses, and state these hypotheses clearly.

<u>Hint</u> : Invent a path-independence lemma that applies to linear forms in x, and use it in addition to the familiar one for quadratic forms.