1. What is the minimum value of \[\int_0^\pi [x(t)]^2 \, dt\]
for \(x\) satisfying \(x(0) = 1\) and \(x(\pi) = 0\)?

2. For the time invariant system
\[\dot{x} = Ax + Bu + cv\]
with a known disturbance \(v(t)\), suggest/describe a control that drives the system from \((x_0, 0)\) to \((0, T)\) and minimizes
\[\eta = \int_0^T u(t) \, dt\]
state any needed hypotheses clearly.

3. Complete the proof the necessary conditions Theorem in Lecture Note 3 (page 5, part (c)).

4. Consider the problem of finding an optimal control for the system:
\[\dot{x} = -ax + u, \quad a > 0;\]
\[\eta = \int_0^1 (x^2 + u^2) \, dt + x^2(1)\]
with initial \(x(0) = 1\).
Use a numerical approach based on canonical equations (and MATLAB) to investigate this problem:
What is the optimal value \(\eta_{\text{min}}\)?