In this course, we cover the basics of the theory of optimization of continuous time, finite dimensional, control systems. We begin with the problem of linear systems with quadratic cost functionals, solving it by use of the matrix Riccati system. We then sketch the elements of calculus in Banach spaces to state and prove the basic theorems of optimization under equality constraints. We follow this with a discussion of optimization in the presence of inequality constraints. This is followed by an introduction to the calculus of variations and its relation to problems of optimal control. This leads to questions of numerical computation of optimal solutions, and we develop the basic algorithms of the subject – gradient and Newton methods. We then derive the second order necessary conditions of the calculus of variations as a point of departure for the intuitive development of Pontryagin’s maximum principle (PMP) that governs the necessary conditions for an optimal control. Finally we give an outline of the theory of sufficient conditions and the Hamilton Jacobi Bellman equations. Through various stages in the course optimal controls will be discussed in open loop and feedback forms, and the relationships between these forms will be highlighted. Connections to Lagrangian and Hamiltonian mechanics will be brought out.

Optional Topics: Numerical methods (unconstrained; constrained; linear programming; interior point methods), calculus of variations and integrable problems of optimal control, solution to certain combinatorial optimization problems by differential equations of gradient type, singular optimal control and higher order necessary conditions.

Course Prerequisite: Math 410 and ENEE 660 (can be waived by instructor consent). Topic Prerequisite: Advanced calculus (at least Math 410 or equivalent; Math 411 preferred); linear system theory (mostly time-domain aspects).


Grading: Weekly homework sets will be collected and graded. There will be two mid-term examinations and a final examination: (i) first mid-term on Tuesday, March 3 (in class, closed book); (ii) second mid-term on Thursday, April 9 (in class, closed book); (iii) final examination during Thursday May 14 – Sunday May 17 (take home, open book, no discussion).

The breakdown in weighting towards the final grade will be: homework 10%, mid-terms 25% each, and finals 40%.