1. For curve $\gamma: [t_0, t_f] \rightarrow \mathbb{R}^3 \rightarrow \mathbf{r}(t)$ show that curvature and torsion take the form

\[ \kappa = \frac{\| \mathbf{\dddot{r}} \times \mathbf{\dot{r}} \|}{\| \mathbf{\dot{r}} \|^3} \]

\[ \tau = \frac{\mathbf{\dot{r}} \cdot (\mathbf{\dddot{r}} \times \mathbf{\dot{r}})}{\| \mathbf{\dot{r}} \times \mathbf{\dddot{r}} \|^2} \]

2. Show that the collection of matrices of the form

\[
\begin{pmatrix}
1 & a_{12} & a_{13} & 0 \\
0 & 1 & a_{23} & 0 \\
0 & 0 & 1 & a_{34} \\
0 & 0 & 0 & a_{44}
\end{pmatrix}
\]

with $a_{44} \neq 0$ is a matrix (Lie) group. Determine a basis for its Lie algebra, and associated structure constants.
5. Show that the smallest Lié algebra of matrices which contains the matrices $A_1, A_2$:

\[ A_1 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} ; \quad A_2 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \]

is four dimensional.

4. Let $t \mapsto \Phi(t)$ be a smooth curve in $\text{Sl}(n; \mathbb{R})$, show that we can write

\[ \dot{\Phi}(t) = \frac{d}{dt} \Phi(t) \xi(t) \]

where $\xi(t)$ has zero trace and $t$.

5. Let $t \mapsto \Phi(t)$ be a smooth curve in $\text{SE}(n; \mathbb{R})$ the special Euclidean group of matrices of the form

\[ \begin{pmatrix} A & b \\ 0 & 1 \end{pmatrix} \]

where $A \in \text{SO}(n)$, $b \in \mathbb{R}^n$ and there is a row of zeroes below $A$. Show that the associated Lie algebra is made up of matrices of the form

\[ \begin{pmatrix} 0 & \Omega \\ 0 & 0 \end{pmatrix} \]

where $\Omega = -\Omega^T$ and $\eta \in \mathbb{R}^n$.

For $n = 3$ determine all structure constants in a suitable/natural basis for this Lie algebra, denoted as $\text{se}(3; \mathbb{R})$. 