ENEE 661 Spring 2013 Homework 1

due date February 7 (Thursday)

1. For curve \( \gamma : [t_0, t_f] \rightarrow \mathbb{R}^3 \) \( t \mapsto \gamma(t) \)
show that curvature and torsion take the form

\[
\kappa = \frac{\| \dot{\gamma} \times \ddot{\gamma} \|}{\| \dot{\gamma} \|^3}
\]

\[
\tau = \frac{\dot{\gamma} \cdot (\ddot{\gamma} \times \dot{\gamma})}{\| \dot{\gamma} \times \ddot{\gamma} \|^2}
\]

2. Show that the collection of matrices of the form

\[
\begin{pmatrix}
1 & a_{12} & a_{13} & 0 \\
0 & 1 & a_{23} & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & a_{44}
\end{pmatrix}
\]

with \( a_{44} \neq 0 \) is a matrix (Lie) group. Determine a basis for its Lie algebra, and associated structure constants.
5. Show that the **smallest** Lie algebra of matrices which contains the matrices $A_1, A_2$:

$$A_1 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}; \quad A_2 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

is **four** dimensional.

4. Let $t \mapsto \Phi(t)$ be a smooth curve in $SL(n; \mathbb{R})$. Show that we can write

$$\dot{\Phi}(t) = \Phi(t) \xi(t)$$

where $\xi(t)$ has **zero** trace $\xi$.

5. Let $t \mapsto \Xi(t)$ be a smooth curve in $SE(n; \mathbb{R})$, the special Euclidean group of matrices of the form

$$\begin{pmatrix} A & b \\ \mathbf{0} & 1 \end{pmatrix}$$

where $A \in SO(n)$, $b \in \mathbb{R}^n$, and there is a row of zeros below $A$. Show that the associated Lie algebra is made up of matrices of the form

$$\begin{pmatrix} 0 & \Omega & \gamma \\ \mathbf{0} & 0 & \mathbf{0} \end{pmatrix}$$

where $\Omega = -\Omega^T$ and $\gamma \in \mathbb{R}^n$.

For $n=3$ determine all structure constants in a suitable/natural basis for this Lie algebra, denoted as $se(3; \mathbb{R})$. 