1. Show that the collection of all $4 \times 4$ matrices of the form

\[
\begin{bmatrix}
1 & a_{12} & a_{13} & 0 \\
0 & 1 & a_{23} & 0 \\
0 & 0 & 1 & a_{44} \\
0 & 0 & 0 & a_{44}
\end{bmatrix}
\]

with $a_{44} \neq 0$ is a matrix Lie group. Determine a basis for its Lie algebra. In your choice of basis, determine the structure constants.

2. Show that the smallest Lie algebra of matrices which contains $A_1$, $A_2$ with

\[
A_1 = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}
\]

\[
A_2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}
\]

is four-dimensional.

3. Let $t \mapsto \Phi(t)$ be a curve in $SL(n, \mathbb{R})$. Show that we can write

\[
\Phi(t) = \Phi(t) \Xi(t)
\]
where $\xi(t) \in sl(n, \mathbb{R})$ i.e. the space of $n \times n$, matrices of trace $= 0$.

4. Let $t \mapsto F(t)$ be a smooth curve in $SE(n, \mathbb{R})$ the Euclidean group of matrices of the form

$$
\begin{bmatrix}
A & b \\
0 & 1
\end{bmatrix}
$$

where $A \in SO(n)$ be $\mathbb{R}^n$ and we have a row of $n$ zeros beneath the matrix $A$. Show that the associated Lie algebra is made up of matrices of the form

$$
\begin{bmatrix}
\Omega & \eta \\
0 & 0
\end{bmatrix}
$$

where $\Omega = -\Omega^T$ and $\eta \in \mathbb{R}^n$.

For $n = 3$ determine all structure constants in a suitable/natural basis for this Lie algebra, denoted as $\mathfrak{se}(3, \mathbb{R})$. 