One of the basic results of single variable calculus is the **mean value theorem (MVT)**

Let \( f : \mathbb{R} \to \mathbb{R} \) be continuous on \([a, b]\) and differentiable on \((a, b)\). There is \(c \in (a, b)\) such that the derivative

\[
f'(c) = \frac{f(b) - f(a)}{b - a}
\]

The adjoining picture gives us an idea of what's going on. The essential geometric idea is that at \(c\) (and \(c'\)) the tangent to the graph of \(f\) is parallel to the line joining points \((a, f(a))\) and \((b, f(b))\).

Let us see what happens in higher dimensions. Consider \(f : [a, b] \to \mathbb{R}^2\)

\[
x \mapsto f(x) = (y, z)
\]

If the curve defined by \(f\) is of the "corkscrew" variety then, there is no \(c \in (a, b)\) at which

\[
a < c < b
\]
Let $a < c < b$. If $f:[a,b] \to \mathbb{R}$

**Lemma**

We need some preliminary results.

The concept from of mean value theorem.

Example:

$f:[0, \pi/2] \to \mathbb{R} \quad f(x) = \sqrt{x}$

& \quad \text{domain must hold. For a specific.}

The point $x = y \neq \pi/2$. The coordinates

The joining line in part A to the tangent to the curve in part B. The
We need the derivative of $h(x)$.

$L(16) - f(c) \leq h(x) - f(x)$.

Thus, $f(c)$ is a local minimum.

**Lemma 3.** Some hypothesis on $f$ and $a$ in some neighborhood of $c$

Proof: (Essentially some argument on $h$ and $c$).

By Lemma 3.

**Lemma 4.**

$$
\int_a^b (x^2 - 2x) \, dx = \frac{b^3}{3} - b^2. 
$$
\[ f : E \rightarrow \mathbb{R} \supset F = R \mid y \]

\[ (1) \quad \text{if } F \in \mathcal{F} \text{ and } \mathcal{F} \in \mathcal{G} \text{ then } \mathcal{G} \in \mathcal{F} \]

**Example:**

- **Clarke Derivative:**

  Sometimes by \( \bar{f}(x) \).

- **Outer Regularity by \( \bar{f}(x) \):**

  a regular function of \( f \) and \( t \).

Let \( t = 0 \) if \( f(x) = f(a + \epsilon t) \) for all \( t \),

\[
\lim_{t \to 0} \frac{f(x + \epsilon t) - f(x)}{t} = 0.
\]

Clearly, \( f \) is \( \bar{C}^0 \) at \( a \) for each \( t \) in \( A \).

Here \( \bar{y} \) is such that \( x \in B \times \bar{y} \).

\[
\lim_{t \to 0} \frac{f(x + \epsilon t) - f(x)}{t} = 0.
\]

Let \( E \rightarrow F \) such that

\[
L : E \rightarrow F \text{ is a continuous linear map.}
\]

If \( f \) is \( \bar{C}^0 \) differentiable at \( x \) and \( L \),

\[
f : U \rightarrow F.
\]

Clearly, \( f \) is \( \bar{C}^0 \) differentiable at \( a \) and \( \mathcal{F} \).

**Definition:**

Let \( E \rightarrow F \) be \( \bar{C}^0 \) homed linear.

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\[ f(x + h) = f(x) + f'(x)h \]

There is a mapping \( u \) from \( \mathbb{R}^n \) to \( \mathbb{R}^n \) such that \( f(u(x)) = f(x) \) for all \( x \) in the domain of \( f \).

Let \( U \) be an open subset of \( \mathbb{R}^n \) and \( f: U \to \mathbb{R}^n \) be a smooth map.

The Fundamental Theorem of Integral Calculus.

Another useful result from calculus is:

\[ \int_u^v f(t) \, dt = \int_u^v f(t) \, dt \]

where \( u, v \) are points in the domain of \( f \).

Prove: Simply compute the integral from \( u \) to \( v \).

There exist \( a, b, c, d \) such that \( a + b + c + d = 0 \) and \( \|f'(a)\| \leq \delta \).

\[ \|f(b) - f(a)\| \leq \delta \|

\[ \text{Conclude the result.} \]

Let \( f: \mathbb{R}^n \to \mathbb{R}^m \) be a smooth map.

Mean Value Theorem.
Let $f : [a, b] \times C \to \mathbb{R}$, for generic $C \in \mathbb{C}$.

Lemma: Consider the function $f(x, y) = x + y$.

Proof. Set $g(t) = f(t^2, t^3)$. The function $g(t)$ is defined for all $t$. We find that $g(t) = t^2 + t^3$. Therefore, $g'(t) = 2t + 3t^2$. Since $g'(t)$ is continuous, we have $g(t)$ is also continuous.
Choose \( x \) and \( y \) such that \( x \neq y \). Then, by the Mean Value Theorem, there exists \( \xi \in [x, y] \) such that

\[
\frac{f(y) - f(x)}{y - x} = f'\left(\xi\right) = \frac{f(e) - f(y)}{e - y}.
\]

Similarly, for \( y \neq z \), there exists \( \eta \in [y, z] \) such that

\[
\frac{f(z) - f(y)}{z - y} = f'\left(\eta\right) = \frac{f(z) - f(e)}{z - e}.
\]

Since \( f' \) is continuous, \( f'' \) is integrable on \([a, b] \times [a, b] \), and \( 0 \leq f''(x, y) \leq 1 \), we have

\[
\int_{a}^{b} \int_{a}^{b} f''(x, y) \, dx \, dy \leq b - a.
\]
\[ x = (x, \frac{xe}{f(x)}) \]

Then \( x = (1, t) \) and

\[ \begin{align*}
xe &= x\frac{xe}{f(x)} \\
xe &= x\frac{xe}{f(x)} \\
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xe &= x\frac{xe}{f(x)}
\end{align*} \]

which in the previous example

\[ (0, 1) = (\vartheta, (1, e)) \]

Determine the stationary value function

\[ (x, \frac{xe}{f(x)}) = (x, \frac{xe}{f(x)}) \]

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Hence let \( g(t) = \int (0, 1) \)

Prove that exists a point \( x \neq \emptyset \) such that

\[ \text{minimum } \int (0, 1) \text{ such that the line } \]

Suppose \( x \neq \emptyset \) are such that the line

at each point \( x \neq \emptyset \) open set \( S \subseteq \mathbb{R}^n \).

Suppose \( f \) is in \( C^1 \), i.e. continuously differentiable.

\[ \text{at } (0, x, 0, \theta) \]