ENEE 660 Fall 2010 Homework 8 [due in class November 9]

1. Find a finite dimensional linear dynamical system that yields a weighting pattern of the form:

   (i) \( T(t, \sigma) = \sin(t) \sin(\sigma) \),
   (ii) \( T(t, \sigma) = \sin(t-\sigma) \).

   For which of these two cases can a time-invariant realization be found?

2. Read Lecture Notes 5 (Floquet Theory) and do Problem 3 in Section 9 page 53 of R.W. Brockett. [This material was sent to you by email]

3. Consider a time-varying scalar system of the form

   \[
   \ddot{x}(t) + ax(t) + bx(t) = u(t) + f(t)u(t)
   \]

   with \( a \) and \( b \) constants and \( f \) differentiable. Find a first order vector differential equation representation of the system. Find the weighting pattern relating \( x \) and \( u \).
4. Show that the weighting pattern associated with the system

\[ \dot{x}(t) = Ax(t) + e^{tF}Bu(t) \]
\[ y(t) = Ce^{-tF}x(t) \]

is stationary if \( FA = AF \). In that case, calculate it.

5. Show that a constant realization \([A, B, B^T]\) with \( A = -A^T \) has a weighting pattern of the form

\[ T(t, \sigma) = G(t)G^T(\sigma) \]