Exercise 5.A


Exercise 5.B

*Hayes*, Problem 4.2.

Problem 5.1

Consider a real-valued process \( x[n] \) for which \( x[0] = 0 \), and which satisfies the first-order AR model equation

\[
x[n] + a[1] x[n-1] = v[n],
\]

where \( a[1] \) is a real-valued constant and \( v[n] \) is a real-valued stationary white noise process with mean \( m_v \) and variance \( \sigma_v^2 \). Clearly, since \( x[0] = 0 \), the process \( x[n] \) is not wide-sense stationary (except in the trivial case that \( x[n] = 0 \) for all \( n \)).

(a) Determine the mean \( m_x[n] \) of the process \( x[n] \) for \( n > 0 \).

(b) Determine whether or not \( x[n] \) is asymptotically stationary in the mean. (A process \( x[n] \) is asymptotically stationary in the mean if the mean \( m_x[n] \) becomes independent of \( n \) as \( n \to \infty \), i.e., if \( \lim_{n \to \infty} m_x[n] \) exists and is finite.)

(c) Determine the variance \( \text{var}(x[n]) \) of the process \( x[n] \) for \( n > 0 \) in the case that \( v[n] \) is zero-mean.

(d) Determine whether or not \( x[n] \) is asymptotically wide-sense stationary in the case that \( v[n] \) is zero-mean. (A process \( x[n] \) is asymptotically wide-sense stationary if its mean \( m_x[n] \) becomes independent of \( n \) and its autocorrelation function \( r_x[n; n-k] \) depends only on the lag \( k \) as \( n \to \infty \).)
Problem 5.2

*Hayes*, Problem 4.5.

Problem 5.3

*Hayes*, Problem 4.7.

Problem 5.4


Problem 5.5

*Hayes*, Problem 4.19.

Problem 5.6

*Hayes*, Problem 4.21.

Problem 5.7

Consider the problem of modeling the finite-duration signal

\[ x[n] = \delta[n] - \alpha\delta[n - 1] \]

via the impulse response of an all pole-model \( A_p(z) \), where the filter parameters in \( A_p(z) \) are to be selected by means of the autocorrelation method. We are interested in the two cases \(|\alpha| < 1\) and \(|\alpha| > 1\).

(a) Determine the set of equations that must be satisfied by the parameters \( a_p[k] \) of the \( p \)-th order all-pole model \( A_p(z) \) selected via the autocorrelation method. In addition, verify that for \(|\alpha| \neq 1\),

\[ a_p[k] = \alpha^k \frac{1 - \alpha^{2(p-k+1)}}{1 - \alpha^{2(p+1)}} \]

for \( k = 1, 2, \ldots, p \) is the solution to these equations. Finally, determine the associated squared error \( \epsilon_p \).

(b) Determine the limit of the all-pole model \( A_p(z) \) and the associated squared error \( \epsilon_p \) as \( p \to \infty \), in the two cases \(|\alpha| < 1\) and \(|\alpha| > 1\).