**ENEE 630 Homework 2**  
**Material Covered:** Polyphase Representation, Uniform DFT Filter Banks,  
Analysis/Synthesis system  

**Problem 1** Consider the structure shown in Fig P-1(a), where \( W \) is the \( 3 \times 3 \) DFT matrix.

This is a three channel synthesis bank with three filters \( F_0(z) \), \( F_1(z) \), \( F_2(z) \). (For example \( F_0(z) = Y(z)/Y_0(z) \) with \( y_1(n) \) and \( y_2(n) \) set to zero.)

a) Let \( R_0(z) = 1 + z^{-1} \), \( R_1(z) = 1 - z^{-2} \), \( R_2(z) = 2 + 3z^{-1} \). Find expressions for the three synthesis filters \( F_0(z) \), \( F_1(z) \), \( F_2(z) \).

b) The magnitude response of \( F_1(z) \) is sketched in Fig. P-1(b). Plot the responses \( |F_0(e^{j\omega})| \) and \( |F_2(e^{j\omega})| \). Does the relation between \( F_0(z) \), \( F_1(z) \), and \( F_2(z) \) depend on choices of \( R_k(z) \)?

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*Figure: P-1(a)*

*Figure: P-1(b)*

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Problem 2 For a uniform DFT analysis bank, we know that the filters are related by $H_k(z) = H_0(zW^k)$, $0 \leq k \leq M - 1$, with $W = e^{-j2\pi/M}$. Let $M = 5$ and define two new transfer functions $G_1(z) = H_1(z) + H_4(z)$ and $G_2(z) = H_2(z) + H_3(z)$. Let $h_0(n)$ denote the impulse response of $H_0(z)$, assumed to be real for all n.

a) Are $h_k(n)$, $1 \leq k \leq 4$ real for all n?
b) Express the impulse responses $g_1(n)$ and $g_2(n)$ of $G_1(z)$ and $G_2(z)$ in terms of $h_0(n)$. Are $g_1(n)$ and $g_2(n)$ real for all n?
c) Let $|H_0(e^{j\omega})|$ be as shown in Fig. P-2. Plot the responses $|G_1(e^{j\omega})|$ and $|G_2(e^{j\omega})|$, for $0 \leq \omega \leq 2\pi$. Does $|G_2(e^{j\omega})|$ necessarily look ‘good’ in its passband?

![Figure : P-2](image_url)

Problem 3 Let $H(z) = \sum_{n=0}^{N} h(n)z^{-n}$ with $h(n) = h(N - n)$. Consider the Type-1 polyphase representation for $H(z)$ with $M$ polyphase components. The symmetry of $h[n]$ reflects into the coefficients of the polyphase components $E_l(z)$ as follows: there exists an integer $m_0$ (with $0 \leq m_0 \leq M - 1$) such that $e_k[n]$ is the image of $e_{m_0-k}[n]$ for $0 \leq k \leq m_0$, and $e_k[n]$ is the image of $e_{M+m_0-k}[n]$ for $m_0 + 1 \leq k \leq M - 1$.

a) Take an example of a $7^{th}$-order $H(z)$ and verify the above statement for $M = 3$. What is $m_0$? How about when $M = 4$?
b) Prove the above statement. Find out how $m_0$ is related to $N$ and $M$. 
**Problem 4** Consider the analysis/synthesis system in Fig. P-4.

a) Let the analysis filters be \( H_0(z) = 1 + 3z^{-1} + 0.5z^{-2} + z^{-3} \) and \( H_1(z) = H_0(-z) \). Find causal stable IIR filters \( F_0(z) \) and \( F_1(z) \) such that \( \hat{x}(n) \) agrees with \( x(n) \) except for a possible delay and (nonzero) scale factor.

b) Let \( H_0(z) = 1 + 2z^{-1} + 3z^{-2} + 2z^{-3} + z^{-4} \), and \( H_1(z) = H_0(-z) \). Find causal FIR filters \( F_0(z) \) and \( F_1(z) \) such that \( \hat{x}(n) \) agrees with \( x(n) \) except for a possible delay and (nonzero) scale factor.

(Hint: Polyphase decomposition may help to solve this problem. Review of complementary filters might help as well. And there is a counter part of Euclid’s theorem in polynomial form for two relatively prime polynomials.)