Problem 1 solution:

\[ y[n] = \frac{1}{2}[1 + (-1)^n]x[n] \]

Problem 2 solution:

Z-transform of \( a^n \) does not converge except for \( a = 0 \), so the FT for \( x[n] = e^{j\omega_0 n} \) does not exist in the usual sense. However, by using a Dirac delta function \( \delta_a(\cdot) \), we can represent its FT as \( 2\pi\delta_a(\omega - \omega_0) \) for \( 0 < \omega < 2\pi \) and repeat it every \( 2\pi \). As a result, the FT of \( \cos(2\pi kn/L) \) can be represented by

\[ \pi\delta_a(\omega - \frac{2\pi k}{L}) + \pi\delta_a(\omega + \frac{2\pi k}{L}) \]

The multiplication \( x[n] \) and \( y[n] \) results in the integral of the convolution of \( X(\omega) \) and \( Y(\omega) \) over \( 2\pi \) in the frequency domain. In Problem 2, since \( h_k[n] = h_0[n]cos(2\pi kn/L) \), we have

\[
H_k(\omega) = \frac{1}{2\pi} \int_{2\pi} H_0(\omega) \ast [\pi\delta_a(\omega - \frac{2\pi k}{L}) + \pi\delta_a(\omega + \frac{2\pi k}{L})]d\omega = \frac{1}{2}[H_0(\omega - \frac{2\pi k}{L}) + H_0(\omega + \frac{2\pi k}{L})]
\]

Thus, \( Y_0(\omega) = \frac{1}{2}[H_0(\omega - \frac{2\pi k}{L}) + H_0(\omega + \frac{2\pi k}{L})]X(\omega) \), which gives

\[
Y_0(\omega) = \frac{1}{2}[H_0(\omega - \frac{2\pi k}{L}) + H_0(\omega + \frac{2\pi k}{L})]X(\omega L)
\]
On the other hand, $U(\omega) = H_0(\omega)X(\omega L)$, and $Y_1(\omega)$ is given by

$$Y_1(\omega) = \frac{1}{2}[U(\omega - \frac{2\pi k}{L}) + U(\omega + \frac{2\pi k}{L})]$$

Combine the previous two relations, we arrive at

$$Y_1(\omega) = \frac{1}{2}[H_0(\omega - \frac{2\pi k}{L}) + H_0(\omega + \frac{2\pi k}{L})]X(\omega L)$$

which is the same as $Y_0(\omega)$.

**Problem 3 solution:**

Following the discussion in Problem 2,
\[
Y_0(\omega) = H_k(\omega)Y(\omega) = \frac{1}{2}[H_0(\omega - \frac{2\pi k}{L}) + H_0(\omega + \frac{2\pi k}{L})]Y(\omega)
\]

\[
Y_1(\omega) = \frac{1}{2}[H_0(\omega - \frac{2\pi k}{L})Y(\omega - \frac{2\pi k}{L}) + H_0(\omega + \frac{2\pi k}{L})Y(\omega + \frac{2\pi k}{L})]
\]

They are not equal in general.

**Problem 4 solution:**

\[x(n) \xrightarrow{H(z)} \downarrow 2 \xrightarrow{H(z)} \downarrow 2 \xrightarrow{s(n)} \uparrow 4 \rightarrow y(n)\]

![Diagram showing signal processing steps](image)

ver. 201210