Problem 1 For the system in Fig. P-1, find an expression for \( y(n) \) in terms of \( x(n) \). Simplify the expression as best as you can.

\[
x(n) \xrightarrow{\uparrow 3} y(n) \xrightarrow{\downarrow 2} x(n) \xrightarrow{\downarrow 3} y(n) \xrightarrow{\uparrow 2}
\]

Figure : P-1

Problem 2 Show that the two systems shown in Fig. P-2(a) (where \( k \) is some integer) are equivalent (that is, \( y_0(n) = y_1(n) \) ) when \( h_k(n) = h_0(n)\cos(2\pi kn/L) \).

\[
x(n) \xrightarrow{\uparrow L} y(n) \xrightarrow{H_k(z)} y_0(n) \quad x(n) \xrightarrow{\uparrow L} y(n) \xrightarrow{H_0(z)} u(n) \xrightarrow{\cos \frac{2\pi kn}{L}} y_1(n)
\]

Figure : P-2(a)

This is a structure where filtering followed by cosine modulation has the same effect as filtering with the cosine modulated impulse response. (This is not true in all situations; see next problem). Now consider the example where \( L = 5 \), and \( k = 1 \). Let \( X(e^{j\omega}) \) and \( H_0(e^{j\omega}) \) be as sketched in Fig. P-2(b). Give sketches of \( Y(e^{j\omega}), Y_0(e^{j\omega}) \) and \( U(e^{j\omega}) \).

\[
0 \quad 2\pi \quad \omega
\]

\[
0 \quad \pi/5 \quad 2\pi \quad \omega
\]

Figure : P-2(b)
Problem 3 Show that the two systems shown in Fig.P-3 are not equivalent, that is, \( y_0(n) \) and \( y_1(n) \) are not necessarily the same, even if \( h_k(n) = h_0(n)\cos(2\pi kn/L) \).

\[
\begin{align*}
\text{Problem 4} & \quad \text{Consider a sequence } x(n) \text{ with } X(e^{j\omega}) \text{ as shown in Fig. P-4(a).} \\
& \quad \text{Suppose we generate the sequences } y(n) \text{ and } s(n) \text{ from } x(n) \text{ as in Fig. P-4(b), where} \\
& \quad H(e^{j\omega}) = \begin{cases} 
1 & \text{for } |\omega| < \pi/2 \\
0 & \text{for } \pi/2 \leq |\omega| \leq \pi
\end{cases} \\
& \quad \text{Plot the quantities } Y(e^{j\omega}) \text{ and } S(e^{j\omega}).
\end{align*}
\]