1. Consider the scalar quantization of a \( \mathcal{R} \)-valued rv. \( X \) with \( E[|X|] < \infty \), \( E[|X|^2] < \infty \). Let a quantizer \( Q : \mathcal{R} \rightarrow \mathcal{C} = \{y_1, ..., y_N\} \subset \mathcal{R} \) be such that its codebook \( \mathcal{C} \) satisfies the "centroid" condition. Show that
   a) \( E[X - Q(X)] = 0 \), i.e., the quantizer output is unbiased;
   b) \( E[Q(X)(X - Q(X))] = 0 \), i.e., the quantizer output is uncorrelated with the quantization error; and
   c) \( E[(X - Q(X))^2] = \sigma_X^2 - \sigma_{Q(X)}^2 \), i.e., the variance of the quantization error equals the difference of the variances of the signal \( X \) and its quantized version.

2. Proakis, Problem 4.19: \( \pi/4 \)-QPSK may be considered as two QPSK systems offset by \( \pi/4 \) radians.
   a) Sketch the signal space diagram for a \( \pi/4 \)-QPSK signal.
   b) Using Gray encoding, label the signal points with the corresponding data bits.

3. Proakis, Problem 4.21 (a), (c): A PAM partial response signal (PRS) is generated as shown in Figure P4.21 by exciting an ideal low-pass filter of bandwidth \( W \) by the sequence \( B_n = I_n + I_{n+1} \) at a rate \( \frac{1}{T} = 2W \) symbols/s. The sequence \( \{I_n\} \) consists of binary digits selected independently from the alphabet \( \{+1, -1\} \) with equal probability. Hence, the filtered signal has the form
   \[
   v(t) = \sum_{n=-\infty}^{\infty} B_n g(t - nT), \quad T = \frac{1}{2W}
   \]
   a) Sketch the signal space diagram for \( v(t) \) and determine the probability of occurrence of each symbol.
   b) Determine the autocorrelation and power density spectrum of the three-level sequence \( \{B_n\} \).
   c) The signal points of the sequence \( \{B_n\} \) form a Markov chain. Sketch this Markov chain and indicate the transition probabilities among the states.

4. Proakis, Problem 4.22: The low-pass equivalent representation of a PAM signal is
   \[
   u(t) = \sum_n I_n g(t - nT)
   \]
   Suppose \( g(t) \) is a rectangular pulse and
   \[
   I_n = a_n - a_{n-2}
   \]
where \( \{ a_n \} \) is a sequence of uncorrelated binary-valued (1,-1) random variables that occur with equal probability.

a) Determine the autocorrelation function of the sequence \( \{ I_n \} \).
b) Determine the power density spectrum of \( u(t) \).
c) Repeat (b) if the possible values of the \( a_n \) are \( (0,1) \).

5. Proakis, Problem 4.28 (just the phase tree): Sketch the phase tree, the state trellis, and the state diagram for partial response CPM with \( h = \frac{1}{2} \) and

\[
g(t) = \begin{cases} 
\frac{1}{T} & 0 \leq t \leq 2T \\
0 & \text{otherwise}
\end{cases}
\]

6. Proakis, Problem 4.30: Show that 16 QAM can be represented as a superposition of two four-phase constant envelope signals where each component is amplified separately before summing, i.e.,

\[
s(t) = G(A_n \cos 2\pi f_c t + B_n \sin 2\pi f_c t) + (C_n \cos 2\pi f_c t + D_n \sin 2\pi f_c t)
\]

where \( \{ A_n \}, \{ B_n \}, \{ C_n \}, \) and \( \{ D_n \} \) are statistically independent binary sequences with elements from the set \( \{+1, -1\} \) and \( G \) is the amplifier gain. Thus, show that the resulting signal is equivalent to

\[
s(t) = I_n \cos 2\pi f_c t + Q_n \sin 2\pi f_c t
\]

and determine \( I_n \) and \( Q_n \) in terms of \( A_n, B_n, C_n \) and \( D_n \).

7 Benedetto-Biglieri, Problem 6.5: Derive the squared Euclidean distance \( d_B^2 \) for partial-response CPM with rectangular pulses and \( L = 2 \). Compare the values obtained by considering the merges at \( t = 3T \) and those at \( t = 4T \).
Figure 1: P4.21