ESTIMATION AND DETECTION THEORY

HOMEWORK # 7:

Please work out the ten (10) problems stated below – HVP refers to the text: H. Vincent Poor, An Introduction to Signal Detection and Estimation (Second Edition), Springer Texts in Electrical Engineering Springer, New York (NY), 2010. With this in mind, Exercise II.2 (HVP) refers to Exercise 2 for Chapter II of HVP. Exercises are located at the end of each chapter.

Show work and explain reasoning.

1. Solve Exercise IV.5 (HVP).

2. Solve Exercise IV.6 (HVP).


4. Solve Exercise IV.11 Part (a) (HVP).

5. Solve Exercise IV.12 (HVP).


7. Solve Exercise IV.15 (HVP).

8. Solve Exercise IV.16 (HVP).

9. The independent scalar observations \( Y_1, \ldots, Y_k \) for some positive integer \( k \) all have finite mean \( m \) and variance \( \sigma^2 \)
9.a Consider the estimator \( g_1 : \mathbb{R}^k \to \mathbb{R} \) given by

\[
g_1(y_1, \ldots, y_k) = \frac{1}{k} \sum_{\ell=1}^{k} y_\ell, \quad y_\ell \in \mathbb{R},
\]

Is this a biased estimator of \( m \) on the basis of \( Y_1, \ldots, Y_k \)?

9.b Consider now the estimator \( g_2 : \mathbb{R}^k \to \mathbb{R} \) given by

\[
g_2(y_1, \ldots, y_k) = \frac{1}{k} \sum_{\ell=1}^{k} (y_\ell - g_1(y_1, \ldots, y_k))^2, \quad y_\ell \in \mathbb{R},
\]

Is this an unbiased estimator \( \sigma^2 \) on the basis of \( Y_1, \ldots, Y_k \)?

10. Let \( Y_1, \ldots, Y_n \) be i.i.d. Poisson rvs with parameter \( \theta > 0 \). Show that the statistic \( T : \mathbb{R}^n \to \mathbb{R} \) given by

\[
T(y_1, \ldots, y_n) = \sum_{i=1}^{n} y_i, \quad y_1, \ldots, y_n \in \mathbb{R}
\]

is a complete sufficient statistics for the family \( \{ F^{(\alpha)}_{\theta}, \theta > 0 \} \) where \( F^{(\alpha)}_{\theta} \) is the joint probability distribution of the rv \( (Y_1, \ldots, Y_n)' \) under \( P_{\theta} \).

Find the MVUE for estimating \( \theta \) on the basis of \( (Y_1, \ldots, Y_n)' \). Carefully explain your reasoning!