**Problem 1.** (2.5 pts) Wilson Current Mirrors  

a) For the current mirror on the right, find the expression for the output current in terms of the input current and $\beta$. (Do not assume that $\beta$ is large) Show all of your steps.

If we call the emitter current of Q2, ‘I’, we know from lecture that the bipolar current mirror with identical transistors will produce at the collector of Q3:

$$I_{c_{Q3}} = I \frac{\beta}{\beta + 2}$$

At the output, $I_{out} = \alpha \cdot I$

At the input, $I_{in} = I \left( \frac{1}{\beta + 1} + \frac{\beta}{\beta + 2} \right) = \frac{I_{out}}{\alpha} \left( \frac{1}{\beta + 1} + \frac{\beta}{\beta + 2} \right)$ so, solving for the current gain,

$$\frac{I_{out}}{I_{in}} = \frac{\alpha}{1 + \frac{\beta(\beta + 1)}{(\beta + 2)}} = \frac{\beta}{\beta(\beta + 1) + \beta + 2} = \frac{\beta^2 + 2\beta}{\beta^2 + 2\beta + 2}$$

which is very close to 1.

b) For the current mirror on the right, find the expression for the output current in terms of the input current and $\beta$. (Do not assume that $\beta$ is large) Show all of your steps.

As in part a), if we call the emitter current of Q2, ‘I’, we know from lecture that the bipolar current mirror with identical transistors will produce at the collector of Q3:

$$I_{c_{Q3}} = I \frac{\beta}{\beta + 2}$$

$I_{out} = \alpha \cdot I$

$$I_{in} = \alpha \left( \frac{\beta}{\beta + 2} \cdot I \right) + \frac{\beta}{\beta + 2} \cdot \frac{1}{\beta + 1} \cdot I + \frac{1}{\beta + 1} \cdot I = I \left( \frac{\beta^2}{(\beta + 2)(\beta + 1)} + \frac{\beta}{(\beta + 2)(\beta + 1)} + \frac{\beta + 2}{(\beta + 2)(\beta + 1)} \right)$$

$$I_{in} = I \left( \frac{\beta^2 + 2\beta + 2}{(\beta + 1)(\beta + 2)} \right) = \frac{I_{out}}{\alpha} \left( \frac{\beta^2 + 2\beta + 2}{(\beta + 1)(\beta + 2)} \right) = \frac{I_{out}}{\beta} \left( \frac{\beta^2 + 2\beta + 2}{(\beta + 2)} \right)$$

$$\frac{I_{out}}{I_{in}} = \frac{\beta(\beta + 2)}{\beta^2 + 2\beta + 2}$$

which is the same answer as in part (a)

c) Give an intuitive explanation for the relationship between the two answers for (a) and (b)
We can see that the answers should be the same because the KCL expression for $I_{in}$ produces the same current going into the base of $Q_2$, $\frac{I}{\beta+1}$, and produces the same amount of current going into the collector of $Q_3$, $\frac{\beta}{\beta+2}$, thus all the other currents will be the same and produce the same answer for $I_{out}$.

**Problem 2. (3.5 pts) Transconductance Amplifier (PSPICE)**

We discussed that it is possible to calculate the voltage gain of a circuit by multiplying the output resistance by the transconductance. For the transconductance amplifier on the right, using PSPICE, measure:

To begin with, we find a common-mode voltage for the differential input that will ensure that all transistors are in saturation.

We started with 2.5V because it was about 2 Vgs plus a little more. Solving for the bias point, we check to make sure that the voltages work out. Notice that the drain voltage of $M_5$ is 1.247V > VG5. The bias transistor is clearly in saturation. Notice also that with nothing connected to $V_{out}$, it settles to equal $V_{D3}$. We will choose this voltage to be our zero current output voltage.

**a) the transconductance**

Using the zero-current output voltage 3.713V, we hold $V_{out} = 3.713V$ and measure the current for tiny changes in the differential input around the common-mode voltage 2.5V.

Using the cursors, we measured over 20mV of differential input, the output current changed 466nA. This corresponds to a transconductance = $23.3 \mu A/V$

**b) the output resistance**

Using a zero-input signal (i.e., $V_{in1} = V_{in2} = V_{commonmode} = 2.5V$) we scan the output voltage $V_{out}$ and measure the current that is drawn from the voltage source. While we don’t have to measure this around the
zero current output voltage, this ensures us that we are squarely in saturation with no chance of hitting the nonlinear edges of the range.

Moving the voltage $V_{out}$ over 200mV produced a current that changed over 13.2 nA. This corresponds to an output resistance $= 15.2\, \text{MOhm}$

c) the voltage gain
In this measurement, we need to release $V_{out}$ and let it change as we change the input differential voltage. The amplifier’s voltage gain will only be valid around where $V_{in1} = V_{in2}$, and where $V_{out}$ is such that all the transistors are saturated. We’ll only consider $V_{out}$ between $V_{in2}$ and $VG_3$.

Over a 3.37mV differential input voltage range, the output voltage moved 1.20V.
If we define $V_{in1} - V_{in2}$ as the differential input voltage, then we have a positive gain $= 356$

d) the ratio of the voltage gain (measured) / voltage gain (calculated using (a) and (b)).

The voltage gain computed from the transconductance and output resistance is: $(23.3\mu\text{A/V})(15.2\, \text{M}) = 354$
Wow! The ratio is: $356/354 = 1.00565$

Problem 3. (4.5 pts) Gate-Source Referenced Current Source (PSPICE)
We discussed the gate-source referenced current sources for the bipolar and the MOSFET in class. We identified this circuit as being good for rejecting fluctuations on the power supply.
a) Find the expression for the small signal output resistance in terms of our parameter variables (e.g., \(k_n'\), \(g_m\), \(g_{mb}\), etc.)

\[
\frac{-V_1}{r_{o2}} = g_{m2} \cdot V_X \quad \text{and} \quad i_i = \frac{V_X}{100K}
\]

KCL at x gives:

\[
(v_1 - v_x)(g_{m1} + (-v_x)g_{mb1} + \frac{v_t - v_x}{r_{o1}}) = \frac{V_x}{100K}
\]

By plugging in for \(v_i\), separating out the \(v_x\), and solving for \(v_t\) in terms of \(v_x\), we get:

\[
-\left(-g_{m2} \cdot r_{o2} \cdot v_x - v_x\right)g_{m1} + (-v_x)g_{mb1} + \frac{-v_x}{r_{o1}} = \frac{-v_i}{100K}
\]

\[
-\left(\left(g_{m2} \cdot r_{o2} + 1\right)g_{m1} + g_{mb1} + \frac{1}{r_{o1}} + \frac{1}{100K}\right) = \frac{-v_i}{r_{o1}} \quad \text{so...}
\]

\[
r_{o1} \cdot V_1 \cdot \left(\left(g_{m2} \cdot r_{o2} + 1\right)g_{m1} + g_{mb1} + \frac{1}{r_{o1}} + \frac{1}{100K}\right) = V_t
\]

since we are interested in finding the output resistance

\(R_o\) which is equal to \(\frac{v_t}{i_t}\)

\[
\frac{V_X}{100K} = \frac{-v_i}{r_{o1}} \cdot \frac{-v_i}{r_{o1}} \cdot \left(\left(g_{m2} \cdot r_{o2} + 1\right)g_{m1} + g_{mb1} + \frac{1}{r_{o1}} + \frac{1}{100K}\right) = \frac{-v_i}{i_t} = R_o \quad \text{This should end up being a large number!}
\]

b) Lookup the MOSIS parameters to find \(k_n'\) and \(V_t\) and estimate what the DC currents will be.

From the MOSIS parameters we find that \(\mu C_{ox}/2 = 57\mu A/V^2\)

From the MOSIS parameters we estimate the nFET threshold voltage to be about 0.7V.

The current on the left half of the circuit, \(I_{D2}\), is clearly going to be 10\(\mu A\).

The current on the right half of the circuit, however, is given by:

\[
I_{D1} = \frac{V_{gs2}}{100K} = \frac{V_t + \frac{2I_{D2}}{k_n'(W/L)}}{100K} = 0.7 + \frac{2 \cdot 10\mu A}{114\mu A \cdot (1)} = 11.2\mu A
\]

c) Using PSPICE, measure the transconductance of equivalent nFETs at these currents, the output resistances \(r_{o1}\) and \(r_{o2}\) at the appropriate current levels. (Note that you can measure the combined effect of \(g_m\) and \(g_{mb}\) by measuring the transconductance of the MOSFET at the appropriate source voltage). Ignore the contribution of \(g_{mb1}\) (i.e., \(g_{mb1} = 0\)). Show printouts of all circuits used to estimate relevant parameters and clearly indicate how you measured each of these values. Now estimate what the output resistance should be based on your equation in (a).
When we run the PSPICE simulation we obtain an output current of 11.61µA, quite close to our calculated estimate.
VG1 = 2.67V
VG2 = 1.161V

For M2, we have 10µA flowing through the drain. The gate voltage at this current level is = 1.161V. So we run a simulation to find the derivative of the current wrt the gate voltage at this operating point. The drain voltage is 2.67V.

Our measurements indicate a $g_{m2} = -178nA / -4.35mV = 40.9\mu A/V$

By sweeping the drain voltage with fixed gate voltage
We observe a change of 1.865nA in drain current over a change in the drain voltage of 17mV
This corresponds to an output resistance $r_{o2} = 9.1\ MOhm$

For M1, we have 11.61µA,

Moving the gate voltage V1 and measuring the drain current we can estimate the $g_{m1}$ that includes the body effect.

Over 17.5mV, we see a change of 778nA, which corresponds to $g_{m1} = 44.5\mu A/V$

Now, measuring the output resistance $r_{o2}$, we move the drain voltage V3 and measure the change in current.
Here we see a change of 24.6nA over a change of 195mV, or $r_0 = 7.9 \text{ MOhm}$

$g_{m1} = 44.5 \mu\text{A/V}$
$g_{m2} = 40.9 \mu\text{A/V}$
$r_0 = 7.9 \text{ MOhm}$
$r_0 = 9.1 \text{ MOhm}$

$$100K \cdot r_0 \cdot \left[ (g_{m2} \cdot r_{02} + 1)g_{m1} + g_{m01} + \frac{1}{r_{01}} + \frac{1}{100K} \right] = \frac{V_i}{I_i} = R_o$$

$$100K(7.9M) \left[ ((40.9\mu)(9.1M) + 1)(44.5\mu) + 0 + \frac{1}{7.9M} + \frac{1}{100K} \right] = 13.1 \text{ GOhm}$$

d) Measure the output resistance from your simulation. Be sure to show a printout of the circuit you used to measure this and **clearly indicate** how you measured this from the simulation.

The PSPICE simulation showed a very flat plot of current as a function of the output voltage $V_{out}$. From the plot using the cursors, a 76.4pA increase in current over a 1.00V change in $V_{out}$ (from 4V to 5V) was measured. This corresponds to an output resistance of **13.1 GOhms**.

d) Compare your measured value vs the computed value for output resistance.

Our measurement of the output resistance and the resistance calculated from measured parameters is surprisingly matched. Although we set $g_{mb1}$ to zero, it contributes so little to the final answer as you can see from the equation. The term $g_{m1} \cdot g_{m2} \cdot R_{02}$ is dominant inside the brackets.
**Problem 4.** (2 pts) Current/Voltage Reference (DESIGN / PSPICE)

In this problem we have a circuit (on the right) that is made up of two building blocks: a current mirror and a gate-source referenced current source. By using the current mirror to make the current source’s input equal to its output, there are basically two solutions: zero current and some positive current. Page 309 in your text (Gray, Hurst, Lewis, and Meyer) describes this circuit.

a) If we want a current of $100\mu A$, find the value $R_1$ that is needed. Explain all of the steps needed to find this value. Use a $V_{dd} = 9V$.

The output current of the mirror drives the current source circuit M1-M2. M2’s drain current must match the mirror output current, so $I_{D2} = \frac{1}{2} k_n' \frac{W}{L} (V_x - V_t)^2$. We can find the voltage $V_x$ that makes this happen.

$$V_x = \sqrt{\frac{2I_{D2}}{k_n' \frac{W}{L}}} + V_t$$

The current source output is then: $I_{D1} = \frac{V_x}{R_1} = \sqrt{\frac{2k_n' \frac{W}{L}}{R_1}}$. Because the mirror makes $I_{D2} = I_{D1}$, we set them to be the same and equal to our target current. From the MOSIS website we find parameters for the T48X run for the AMI 0.5 um process. $k_n' = \mu C_{ox} = 114 \mu A/V^2$ & $W/L = 1$ & $V_t$ about 0.7V

$$R_1 = \frac{\sqrt{\frac{2I_{D2}}{k_n' \frac{W}{L}}} + V_t}{I_{D1}} = \frac{\sqrt{\left(\frac{2 \cdot (10\mu A)}{(114 \mu A/V^2)}\right)} + 0.7V}{10\mu A} = 112 Kohms$$

This is a somewhat awkwardly large value to have to use.

b) Verify your solution using PSPICE. You may need to start with an initial condition somewhere to avoid the zero current solution.
c) Now add a 50mV amplitude sinusoidal voltage source to Vdd and calculate the % change in current given the % change in Vdd.

(See attached plot for Problem 4)

Although the current mirror (in saturation) is supposed to provide the same current on the output as the input, we know that this won’t happen exactly. We pick the output of the mirror as our measurement point because it represents what the current would be if we mirrored out a current using a pFET mirror.

From the simulation we can see that the reference current does fluctuate with the power supply voltage.

We see, for a peak-to-peak voltage change of 100mV (amplitude 50mV), a peak-to-peak fluctuation in the reference current of 8.1427nA, (amplitude of 4.07nA)

The average power supply voltage was 9V.
The percentage change in voltage is: 50mV / 9V = 0.56% change

The average reference current (when Vdd = 9V) was 10.866µA.
The percentage change in reference current is: 0.037%

The reference only passes on 6.6% of the disturbance on the power supply.