This assignment is meant to be a test/warmup of your prerequisite circuits analysis abilities.

For the NPN bipolar transistor, assume that the \( V_{BE}(on) = 0.7V \).

1a. (2 pts) If \( VCC = 5.00V \), solve for \( IB, IC, \) and \( IE \).
   Assume that \( \beta = 100 \). Be sure to justify each step that you take.

   Start by assuming forward active mode. This means that we can use \( Ic = \beta Ib \).

   With \( Vcc = 5V \), we can use \( V_{BE(on)} = 0.7V \), (i.e., \( V_B = 0.7V \)) thus
   \[ I_B = \frac{5 - 0.7}{10K} = 0.43mA \]
   \[ I_C = (100)(0.43mA) = 43.0mA \]
   \[ I_E = I_C + I_B = 43.0mA + 0.43mA = 43.4mA \]
   \[ V_C = 5 - (100)(43.0mA) = 0.7V \] which is > 0.2V thus, the NPN is in forward active mode.

   Barely.

1b. Resolve this using a \( \beta = 200 \)

   Start by assuming forward active mode. This means that we can use \( Ic = \beta Ib \).

   With \( Vcc = 5V \), we can use \( V_{BE(on)} = 0.7V \), (i.e., \( V_B = 0.7V \)) thus
   \[ I_B = \frac{5 - 0.7}{10K} = 0.43mA \]
   \[ I_C = (200)(0.43mA) = 86.0mA \]
   \[ I_E = I_C + I_B = 86.0mA + 0.43mA = 86.4mA \]
   \[ V_C = 5 - (100)(86.0mA) = -3.6V \] which is < 0.2V thus, forward-active was not a good assumption here. We need to resolve this in saturation.

   If we are in saturation, \( V_C = 0.2V \)
   \[ V_B = 0.7V \]
   \[ I_C = \frac{5 - 0.2}{100} = 48mA \]
   \[ I_B = \frac{5 - 0.7}{10K} = 0.430mA \]
   \[ I_E = I_C + I_B = 48.4mA \]

2a. (2 pts) If \( VCC = 5.00V \), solve for \( IB, IC, \) and \( IE \). Assume that \( \beta = 100 \). Be sure to justify each step that you take.

   Start by assuming forward active mode. Because the current in the emitter will be \((1+\beta)\) times that in the base, the 5.6K resistor will look \((1+\beta)\) times larger.
   \[ I_B = \frac{5.00 - 0.7}{10.0K + (101)(5.60K)} = 7.47\mu A \]
\[ I_C = \beta I_B = (100)(7.47 \mu A) = 747 \mu A \]
\[ V_C = 5 - (4.70K)(747 \mu A) = 1.49V \]
\[ V_E = (5.6K)(747 \mu A + 7.47 \mu A) = 4.23V \]

ACK! – This is not consistent with the forward active mode. \( V_C \) should not be lower than \( V_E \). Resolve using the saturation mode.

In saturation, \( I_E = I_C + I_B \) is still true.

\[
\begin{align*}
\frac{V_E}{5.6K} & = \frac{5 - (V_E + 0.2)}{4.7K} + \frac{5 - (V_E + 0.7)}{10K} \\
V_E & = \frac{1}{5.6K} \left( \frac{1}{4.7K} + \frac{1}{10K} \right) = \frac{5 - 0.2}{4.7K} + \frac{5 - 0.7}{10K} \\
V_E(0.491E-3) & = 1.45E-3 \\
V_E & = 2.95V, \ V_B = V_E + 0.7 = 3.65V, \text{ and } V_C = V_E + 0.2 = 3.15V \\
I_E & = \frac{V_E}{5.6K} = 527 \mu A, \ I_B = \frac{5 - V_B}{10K} = 135 \mu A, \ I_C = \frac{5 - V_C}{4.7K} = 393 \mu A
\end{align*}
\]

2b. Resolve this using a \( \beta = 10 \)

Start by assuming forward active mode. Because the current in the emitter will be \((1+\beta)\) times that in the base, the 5.6K resistor will look \((1+\beta)\) times larger.

\[
\begin{align*}
I_B & = \frac{5.00 - 0.7}{10.0K + (11)(5.60K)} = 60.1 \mu A \\
I_C & = \beta I_B = (10)(60.1 \mu A) = 601 \mu A \\
V_C & = 5 - (4.70K)(601 \mu A) = 2.18V \\
V_E & = (5.6K)(601 \mu A + 60.1 \mu A) = 3.70V \\
\end{align*}
\]

ACK! – again… this is also in saturation.

\[
\begin{align*}
\frac{V_E}{5.6K} & = \frac{5 - (V_E + 0.2)}{4.7K} + \frac{5 - (V_E + 0.7)}{10K} \\
V_E & = \frac{1}{5.6K} \left( \frac{1}{4.7K} + \frac{1}{10K} \right) = \frac{5 - 0.2}{4.7K} + \frac{5 - 0.7}{10K} \\
V_E(0.491E-3) & = 1.45E-3 \\
V_E & = 2.95V, \ V_B = V_E + 0.7 = 3.65V, \text{ and } V_C = V_E + 0.2 = 3.15V \\
I_E & = \frac{V_E}{5.6K} = 527 \mu A, \ I_B = \frac{5 - V_B}{10K} = 135 \mu A, \ I_C = \frac{5 - V_C}{4.7K} = 393 \mu A
\end{align*}
\]

Same answer!

3. (2 pts) If \( VCC = 5.00V \), and the diode is known to pass 1mA when \( V_{diode} = 0.55 \), solve for \( V1 \) and \( V2 \) using the exponential model of the diode.

In this problem we are given a current at a known diode voltage. This allows us to calculate the \( I_s \) value for the diode.

\[
1mA = I_s \cdot e^{\frac{0.55}{25mV}} = I_s(1.538_{e9})
\]

\[
I_s = 0.650e^{-12}
\]

To solve this, we will use the iterative method. To solve for the current, we will
lump the resistances together and once we have the current, we will treat them separately to find V1 and V2.

We begin with an estimate for the diode voltage 0.700V.

Thus, \( I_r = \frac{5 - 0.7}{2K} = 2.15mA \) if this were true, then the diode voltage should be:

\[
V_D = (0.026) \ln \frac{I_r}{0.650e-12} = 0.570V
\]

this is our new estimate for the diode voltage. We will now iterate in this manner. With this new estimate, \( I_r = \frac{5 - 0.570}{2K} = 2.22mA \)

\[
V_D = (0.026) \ln \frac{2.22mA}{0.650e-12} = 0.571V
\]

which is very close to our last estimate, so we are almost there. Once more, \( I_r = \frac{5 - 0.571}{2K} = 2.22mA \) which is the same as our last time around, so we have converged at our chosen level of precision. We can therefore solve for V1 and V2.

\[
V1 = 5 - (2.22mA)(1K) = 2.78V
\]

\[
V2 = (2.22mA)(1K) = 2.22V
\]

4. (2 pts) Derive the transfer function \( \frac{v_b}{v_a} \) for the circuit on the right.

We begin by labeling the node connecting R1 and C1, Vx.

If we lump C2 and R2 together as: \( Z_1 = R_2 + \frac{1}{sC_2} = \frac{1+sR_2C_2}{sC_2} \)

\[
V_X = v_a \cdot \frac{Z_1}{R_1 + Z_1} \cdot \frac{1}{sC_1} \quad \text{and} \quad v_b = \frac{R_2}{R_2 + \frac{1}{sC_2}} \cdot V_X
\]

therefore, we can put this altogether as:

\[
v_b = \frac{R_2}{R_2 + \frac{1}{sC_2}} \cdot \frac{Z_1}{R_1 + Z_1} \cdot v_a
\]

\[
v_b = \frac{R_2}{R_2 + \frac{1}{sC_2}} \cdot \frac{1+sC_2R_2}{1+sC_2R_2} \cdot \frac{sC_2+sC_1(1+sC_2R_2)}{R_1+ \frac{1+sC_2R_2}{sC_2+sC_1(1+sC_2R_2)}} \cdot v_a \quad \text{by using} \quad \tau_2 = R_2C_2,
\]

\[
v_b = \frac{1+s\cdot\tau_2}{1+s\cdot\tau_2} \cdot \frac{(sC_2+sC_1)(1+s\cdot\tau_2)}{ \frac{1+s\cdot\tau_2}{sC_2+sC_1(1+s\cdot\tau_2)}} \cdot v_a
\]

\[
v_b = \frac{s\cdot\tau_2}{1+s\cdot\tau_2} \cdot \frac{1+s\cdot\tau_2}{R_1 \cdot (sC_2+sC_1(1+s\cdot\tau_2))+(1+s\cdot\tau_2)} \cdot v_a = \frac{s\cdot\tau_2}{1+s\cdot\tau_2} \cdot v_a
\]
\[ \frac{v_b}{v_a} = \frac{s \cdot \tau_2}{sC_2R_1 + sC_1R_1(1 + s \cdot \tau_2) + (1 + s \cdot \tau_2)} \]

5. (2 pts) According to the Ebers-Moll equations, for what value of \( V_2 \) will the collector current drop to zero? Show your derivation. Simplify your answer to its minimum form. Do not drop the ‘-1’ term. What happens to this ‘zero-current VC’ value if we increase the base voltage?

The relevant Ebers-Moll equation is the collector current equation:

\[ I_C = I_S \left( e^{\frac{V_{be}}{V_T}} - 1 \right) - \frac{I_S}{\alpha_R} \left( e^{\frac{V_{bc}}{V_T}} - 1 \right) = 0 \]

We set it to zero and solve for \( V_{CE} \)

\[ 0 = I_S \left( e^{\frac{V_b}{V_T}} \cdot e^{\frac{-V_c}{V_T}} - 1 \right) - \frac{I_S}{\alpha_R} \left( e^{\frac{V_b}{V_T}} \cdot e^{\frac{-V_c}{V_T}} - 1 \right) \]

\[ 0 = I_S \cdot e^{\frac{V_b}{V_T}} \cdot e^{\frac{-V_c}{V_T}} - I_S \left( e^{\frac{V_b}{V_T}} \cdot e^{\frac{-V_c}{V_T}} - 1 \right) \]

\[ I_S = \frac{I_S}{\alpha_R} \left( e^{\frac{V_b}{V_T}} \cdot e^{\frac{-V_c}{V_T}} - 1 \right) \]

\[ \frac{\alpha_R - 1}{\alpha_R} \cdot e^{\frac{V_b}{V_T}} = e^{\frac{V_b}{V_T}} - 1 \]

with \( V_E = 0 \), \( \frac{\alpha_R - 1}{\alpha_R} \cdot e^{\frac{-V_b}{V_T}} = 1 - \frac{1}{\alpha_R} \cdot e^{\frac{-V_c}{V_T}} \)

\[ \ln \left( \left( 1 - \alpha_R \right) \cdot e^{\frac{-V_b}{V_T}} + \alpha_R \right) = \frac{-V_c}{V_T} \]

Raising the base voltage slightly raises the collector voltage at which the current is zero.