Solutions to Chapter 4 Exercises

4.1. Under the scaling transformation, \( W \rightarrow W/\kappa, L \rightarrow L/\kappa, \tau_{ac} \rightarrow \tau_{ac}/\kappa, V_{ds} \rightarrow V_{ds}/\kappa, V_{g} \rightarrow V_{g}/\kappa, \) and \( V_{t} \rightarrow V_{t}/\kappa, \) Eq. (3.19) becomes

\[
I_{ds} \rightarrow \mu_{eff}(\kappa C_{ac}) \frac{W}{\kappa} \left( \frac{V_{g}}{\kappa} - \frac{V_{t}}{\kappa} \right) \frac{V_{ds}}{\kappa} = \frac{I_{ds}}{\kappa},
\]

and Eq. (3.23) becomes

\[
I_{ds} \rightarrow \mu_{eff}(\kappa C_{ac}) \frac{W}{\kappa} \left( \frac{1}{2m} \left( \frac{V_{g}}{\kappa} - \frac{V_{t}}{\kappa} \right)^{2} \right) = \frac{I_{ds}}{\kappa}.
\]

Note that both \( m = 1 + 3\tau_{ac}/W_{dm} \) and \( \mu_{eff} \) which is a function of \( \beta_{eff} \) given by Eq. (3.49) are nearly invariant under constant field scaling.

4.2. Under the same scaling rules as above, Eq. (3.36) becomes

\[
I_{ds} \rightarrow \mu_{eff}(\kappa C_{ac}) \frac{W}{\kappa} \left( m - 1 \right) \left( \frac{kT}{q} \right)^{2} \exp\left( \frac{e(V_{g} - V_{t})/\kappa}{m} \right).
\]

The \( \exp(-qV_{ds}/kT) \) term has been neglected since typically \( V_{ds} \gg kT/q \). In subthreshold, \( V_{g} < V_{t} \) and \( \exp[q(V_{g} - V_{t})/m\kappa T] > \exp[q(V_{g} - V_{t})/m\kappa T] \) (note that \( \kappa > 1 \)), therefore, the subthreshold current increases with scaling faster than \( \kappa I_{ds} \).

If the temperature also scales down by the same factor, i.e., \( T \rightarrow T/\kappa \), then \( I_{ds} \rightarrow I_{ds}/\kappa \), same as the drift current in Exercise 4.1.

4.3. Since the factor \( \mu_{eff}(V_{g} - V_{t})/(mv_{sat}L) \) is invariant under the scaling transformation, \( W \rightarrow W/\kappa, L \rightarrow L/\kappa, \tau_{ac} \rightarrow \tau_{ac}/\kappa, V_{ds} \rightarrow V_{ds}/\kappa, V_{g} \rightarrow V_{g}/\kappa, \) and \( V_{t} \rightarrow V_{t}/\kappa, \) the square-root expression and therefore the fraction in Eq. (3.78) is unchanged after scaling. The saturation current \( I_{dsat} \) of Eq. (3.78) then scales the same way as the fully saturation-velocity limited current, Eq. (3.80), i.e.,

\[
I_{dsat} \rightarrow (\kappa C_{ac}) \frac{W}{\kappa} v_{sat} \left( \frac{V_{g}}{\kappa} - \frac{V_{t}}{\kappa} \right) \frac{I_{dsat}}{\kappa}.
\]
4.9. Let \( V = 0 \) in Eq. (3.12):

\[
|Q_0| = \frac{qN_a}{N_s} \int_{y_s} e^{\frac{qV}{kT}} \mathcal{F}(y) \, dy.
\]

In the subthreshold region, both the band bending \( \psi \) and the electric field \( \mathcal{F} \) are mainly set by the depletion charge. Because of the exponential factor in \( \psi \), the above integration of inversion charge is dominated by that over a thin surface layer within which \( \mathcal{F}(y) \) is essentially constant and can be taken outside the integral. Thus one obtains

\[
Q_i = \frac{kTn_i^2}{\mathcal{E}N_s} e^{q\psi_s/kT},
\]

where \( \mathcal{E} \) is the surface electric field. Note that \( \exp(q\psi_s/kT) \ll \exp(q\psi_s/kT) \) in weak inversion.

This equation is generally valid for nonuniform (vertically) dopings with \( N_a \) being the p-type concentration at the edge of the depletion layer. (Note that the factor \( N_a \) merely reflects the fact in Fig. 2.24 that the band bending \( \psi_s \) is defined with respect to the bands of the neutral bulk region of doping \( N_a \).

4.10. From Eq. (3.8),

\[
I_{d}(y) = \frac{\mu_{eff} W Q_i(y)}{\mathcal{E}} \frac{dV}{dy}.
\]

Current continuity requires that \( I_{d} \) be a constant, independent of \( y \). For a short-channel or a nonuniformly-doped (laterally) MOSFET,

\[
\int_{0}^{t} \frac{dy}{Q_i(y)} = \mu_{eff} W I_{d} \int_{0}^{t} \frac{dV}{dy} = \mu_{eff} W \frac{V_{ds}}{I_{d}}.
\]

Generalizing the result of the last exercise to laterally varying \( \psi \) and \( \mathcal{F} \), one obtains

\[
\frac{V_{ds}}{I_{d}} = \frac{1}{\mu_{eff} W} \int_{0}^{t} \frac{dy}{Q_i(y)} = \frac{N_s}{\mu_{eff} W kTn_i^2} \int_{0}^{t} \mathcal{F}(y) e^{-q\psi_s(y)/kT} \, dy.
\]

Since \( \mathcal{F}(y) \approx [V_{gs} - V_{fb} - \psi_s(y)]/3I_{ds} \) is not a strong function of \( \psi_s \), the exponential factor dominates. This implies that the subthreshold current is controlled by the point of highest barrier (lowest \( \psi_s \)) in the channel.

SOLUTION
Solutions to Chapter 5 Exercises

5.1. From Eqs. (5.3) and (5.4), the average CMOS inverter delay is

\[ \tau = \frac{\tau_n + \tau_p}{2} = \frac{CV_{dd}}{4} \left( \frac{1}{W_n I_{nlat}} + \frac{1}{W_p I_{p Stat}} \right). \]

If the inverter is driving another stage with the same n- to p-width ratio and if both the n- and p-devices have the same capacitance per unit width, the load capacitance \( C \) is proportional to \( (W_n + W_p) \), i.e.,

\[ \tau \propto (W_n + W_p) \left( \frac{1}{W_n I_{nlat}} + \frac{1}{W_p I_{p Stat}} \right) = \frac{1}{I_{nlat}} + \frac{1}{I_{p Stat}} + \frac{W_p / W_n}{I_{nlat}} + \frac{W_n / W_p}{I_{p Stat}}. \]

The delay is a minimum when the n- to p-width ratio is such that the last two terms are equal, i.e., when \( W_p / W_n = (I_{nlat} / I_{p Stat})^{1/2} \). Note that \( \tau_n < \tau_p \) for minimum delay.

5.2. For the following \( RC \) circuit,

\begin{align*}
\text{if the voltage source is switched to } V_{dd} \text{ at } t = 0, \text{ then} \\
V_{dd} = V(t) + RI(t) = V(t) + RC \frac{dV}{dt}.
\end{align*}

With the initial condition \( V(t = 0) = 0 \), the solution for \( V(t) \) is

\[ V(t) = V_{dd} \left( 1 - e^{-t/RC} \right). \]
The energy dissipated in $R$ is

$$E = \int_{0}^{\infty} R I^2 dt = \frac{V_{dd}^2}{R} \int_{0}^{\infty} e^{-\frac{t}{RC}} dt = \frac{1}{2} CV_{dd}^2,$$

which is independent of $R$. The same amount of energy is stored in $C$.

If the voltage source is now switched to 0, one has

$$V(t) + RC \frac{dV}{dt} = 0.$$

With the initial condition $V(t = 0) = V_{dd}$, the solution for $V(t)$ is

$$V(t) = V_{dd} e^{-\frac{t}{RC}}.$$

The energy stored in $C$ is now all dissipated in $R$:

$$E = \int_{0}^{\infty} R I^2 dt = \frac{V_{dd}^2}{R} \int_{0}^{\infty} e^{-\frac{t}{RC}} dt = \frac{1}{2} CV_{dd}^2.$$

5.3. From Eq. (3.21) and the inversion charge expression above Eq. (3.54), the transit time is

$$\tau_w = \frac{WLC_{ox} (V_g - V_T - mV_{dd} / 2)}{\mu_{eff} C_{ox} (W / L) (V_g - V_T - mV_{dd} / 2) V_{dd}} = \frac{L^2}{\mu_{eff} V_{dd}}$$

for a MOSFET device biased in the linear region.

From Eq. (3.23) and the inversion charge expression above Eq. (3.56), the transit time is

$$\tau_w = \frac{(2/3)WLC_{ox} (V_g - V_T)}{\mu_{eff} C_{ox} (W / L) (V_g - V_T)^2 / 2m} = \frac{4mL^2}{3\mu_{eff} (V_g - V_T)}$$

for a long-channel MOSFET biased in saturation.
5.4. From Eq. (3.78) and the inversion charge expression in Problem 3.10, the transit time is

\[
\tau_r = \frac{Q_i}{I_{dd}} = \frac{L}{V_{sat}} \sqrt{\frac{1 + 2\mu\phi (V_g - V_t)/(m\nu_{sat} L)}{1 + 2\mu\phi (V_g - V_t)/(m\nu_{sat} L) - 1}} + 1/3
\]

for a short-channel MOSFET biased in saturation. The limiting value of \( \tau_r \) is \( L/V_{sat} \) when the device becomes fully velocity saturated as \( L \to 0 \).

5.5. The transmission line model of contact resistance in a planar geometry is represented by the distributed network below. The current flows from a thin resistive film (diffusion with a sheet resistivity \( \rho_s \)) into a ground plane (metal) with an interfacial contact resistivity \( \rho_c \) between them (Fig. 5.13).

![Diagram of a transmission line model with contact resistance](image)

Following a similar approach as in Eqs. (5.20)-(5.22), one can write

\[
V(x + dx) - V(x) = \frac{dV}{dx} dx = -I(x)Rdx,
\]

and

\[
I(x + dx) - I(x) = \frac{dI}{dx} dx = -V(x)Gdx.
\]

Here \( R = \rho_s W \) and \( G = W/\rho_c \). From the above two equations, one obtains

\[
\frac{d^2 f}{dx^2} = RGf = \frac{\rho_{sat}}{\rho_c} f,
\]

where \( f(x) = V(x) \) or \( I(x) \).
\[ = qD_{p\phi 1} \frac{d\Delta p_n(x')}{dx'} \bigg|_{x'=0} \]

\[ = -qD_{p\phi 1} \frac{\Delta p_n(-W_{et})}{L_{p\phi 1} \tanh(W_{et} / L_{p\phi 1})}. \] (10)

Equations (9) and (10) give

\[ S_p = \frac{D_{p\phi 1}}{L_{p\phi 1} \tanh(W_{et} / L_{p\phi 1})}. \]

\[ \beta(\omega) = \left[ \frac{1}{\beta_0} + j\omega \left( \tau_p + \frac{(C_{abc} + C_{dsc})}{g_m} + C_{dsc} (R_L + r_s + r_c) \right) \right]^{-1}. \] (1)

In the high-frequency limit, and for short circuit \((R_L = 0)\), (1) becomes

\[ \beta(\omega) \approx \left[ j\omega \left( \tau_p + \frac{(C_{abc} + C_{dsc})}{g_m} + C_{dsc} (r_s + r_c) \right) \right]^{-1}. \] (2)

The magnitude of \(\beta(\omega)\) becomes unity at \(\omega = \omega_T = 2\pi f_T\), or

\[ 1 \approx 2\pi f_T \left( \tau_p + \frac{(C_{abc} + C_{dsc})}{g_m} + C_{dsc} (r_s + r_c) \right). \] (3)

Eq. (6.99) gives

\[ g_m = \frac{qI_c}{kT}. \] (4)

Therefore, (3) gives

\[ \frac{1}{2\pi f_T} = \tau_p + \frac{kT}{qI_c} \left( C_{abc} + C_{dsc} \right) + C_{dsc} (r_s + r_c). \]
6.5. Neglecting parasitic resistances, Eqs. (6.152) and (6.153) give

\[ I_E = -I_{EBO} \left[ \exp \left( \frac{qV_{BE}'}{kT} \right) - 1 \right] - \alpha_R I_C \]  
(1)

and

\[ I_C = -I_{CBO} \left[ \exp \left( \frac{qV_{BC}'}{kT} \right) - 1 \right] - \alpha_R I_E, \]  
(2)

which can be rearranged to give

\[ V_{BE}' = \frac{kT}{q} \ln \left[ 1 - \frac{I_E + \alpha_R I_C}{I_{EBO}} \right] \]  
(3)

and

\[ V_{BC}' = \frac{kT}{q} \ln \left[ 1 - \frac{I_C + \alpha_R I_E}{I_{CBO}} \right]. \]  
(4)

Now,

\[ V_{CE}' = V_C' - V_E' = (V_C' - V_E') + (V_E' - V_{BE}') = V_{BE}' - V_{BC}'. \]  
(5)

Substituting (3) and (4) into (5) and rearranging, we obtain

\[ V_{CE}' = \frac{kT}{q} \ln \left[ \frac{I_{CBO} (I_{EBO} - I_E - \alpha_R I_C)}{I_{EBO} (I_{CBO} - I_C - \alpha_R I_E)} \right]. \]  
(6)

Now, Eq. (6.90) gives

\[ \alpha_R I_{R0} = \alpha_F I_{RO}, \]  
(7)

and Eqs. (6.150) and (6.151) give

\[ I_{EBO} = I_{RO} (1 - \alpha_R \alpha_F), \]  
(8)

and

\[ I_{CBO} = I_{RO} (1 - \alpha_R \alpha_F). \]  
(9)

Therefore,

\[ \frac{I_{CBO}}{I_{EBO}} = \frac{I_{RO}}{I_{RO}} = \frac{\alpha_F}{\alpha_R}. \]  
(10)

Substituting (10) into (6) gives

\[ V_{CE}' = \frac{kT}{q} \ln \left[ \frac{\alpha_F (I_{EBO} - I_E - \alpha_R I_C)}{\alpha_R (I_{CBO} - I_C - \alpha_R I_E)} \right]. \]
(b) For polysilicon-emitter: $N_E = 10^{20} \text{ cm}^{-3}$ and $W_E = 30 \text{ nm}$. With $\mu_{pE}/\mu_{pB} = 1/3$, we have

$$D_{pE} = \frac{kT\mu_{pE}}{q} = 113 \text{ cm}^2/\text{s}. \quad (5)$$

Now, from Fig. 2.18, we have $L_{pE} = 0.38 \mu\text{m}$, we have

$$L_{pE} = \sqrt[3]{\frac{kT}{q} \mu_{pE} \tau_{pE}^*} = \frac{L_{pE}}{\sqrt{3}} = 0.22 \mu\text{m}. \quad (6)$$

Substituting all these values into (1) we have

<table>
<thead>
<tr>
<th>$W_{E1}$ (nm)</th>
<th>$G_E(30 \text{ nm}, W_{E1})$ (cm$^{-4}$·s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>$1.52 \times 10^{13}$</td>
</tr>
<tr>
<td>100</td>
<td>$2.66 \times 10^{13}$</td>
</tr>
<tr>
<td>200</td>
<td>$4.36 \times 10^{13}$</td>
</tr>
<tr>
<td>300</td>
<td>$5.22 \times 10^{13}$</td>
</tr>
</tbody>
</table>

(c)

![Graph showing the relationship between Polysilicon-layer thickness (nm) and CE(poly)/CE(non-poly)]

7.2. For a box-like profile, Eq. (7.5) gives

$$R_{sE} = \left[ q \int_0^{W_E} P(x, \mu_p(x)) \mu_p(x) dx \right]^{-1} = \frac{1}{q N_B \mu_p W_B} = \frac{\rho_B}{W_B}, \quad (1)$$

where $\rho_B$ is the resistivity of the p-type base layer. Setting $R_{sE} = 10^4 \Omega/\square$, and using Fig. 2.8 for $\rho_B$, we obtain the values in the following table.
7.3. From Exercise 7.2, and Fig. 2.18(a), we have the following table.

<table>
<thead>
<tr>
<th>$N_B$ (cm$^{-3}$)</th>
<th>$\rho_B$ (Ω·cm)</th>
<th>$W_B$ (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6×10$^{16}$</td>
<td>0.3</td>
<td>300</td>
</tr>
<tr>
<td>1×10$^{17}$</td>
<td>0.2</td>
<td>200</td>
</tr>
<tr>
<td>2.9×10$^{17}$</td>
<td>0.1</td>
<td>100</td>
</tr>
<tr>
<td>7×10$^{17}$</td>
<td>0.05</td>
<td>50</td>
</tr>
</tbody>
</table>

![Graph showing base doping vs. base width](image)

where we have used $D_{eb} = kT\mu_{eb}/q$ and $t_b = W_B^2/2D_{eb}$. Also, since Fig. 2.18(a) does not have values for $N_B < 1 \times 10^{17}$ cm$^{-3}$, we have dropped the corresponding base width and base doping concentration values in the table and the following plot.

![Graph showing base transit time vs. base width](image)

SOLUTION
7.4. From Eq. (6.72), we have

\[ V_d = \frac{Q_{ps}}{C_{abc}} = \frac{qN_b W_b}{C_{abc}}. \]  

(1)

To avoid significant base widening, collector current density is kept at

\[ J_c = 0.3 q v_{sat} N_C, \quad \text{or} \quad N_C = \frac{J_c}{0.3 q v_{sat}}, \]  

(2)

with \( v_{sat} = 10^7 \, \text{cm/s}. \)

(a) For a one-sided B-C junction, with \( V_{CB} = 2 \, \text{V}, \) Eqs. (2.68) and (2.70) give

\[ C_{abc} = \frac{\varepsilon_a}{W_{abc}} = \sqrt{\frac{qN_C \varepsilon_a}{2(\psi_{bi} + V_{CB})}} = \sqrt{\frac{J_c \varepsilon_a}{0.6 v_{sat}(\psi_{bi} + V_{CB})}}. \]  

(3)

We can obtain \( \psi_{bi} \) from Fig. 2.12 (or set it to about 0.95 V as an approximation) and use \( \varepsilon_a = 1.04 \times 10^{-12} \, \text{F/cm} \) to generate the following table.

<table>
<thead>
<tr>
<th>( J_c , (\text{mA/\mu m}^2) )</th>
<th>( N_C , (\text{cm}^{-3}) )</th>
<th>( \psi_{bi} , (\text{V}) )</th>
<th>( C_{abc} , (\text{fF/\mu m}^2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>2.08 \times 10^{16}</td>
<td>0.925</td>
<td>0.243</td>
</tr>
<tr>
<td>0.2</td>
<td>4.16</td>
<td>0.945</td>
<td>0.343</td>
</tr>
<tr>
<td>0.3</td>
<td>6.24</td>
<td>0.95</td>
<td>0.420</td>
</tr>
<tr>
<td>0.4</td>
<td>8.32</td>
<td>0.96</td>
<td>0.484</td>
</tr>
<tr>
<td>0.5</td>
<td>1.04 \times 10^{17}</td>
<td>0.97</td>
<td>0.540</td>
</tr>
<tr>
<td>0.8</td>
<td>1.66</td>
<td>0.98</td>
<td>0.659</td>
</tr>
<tr>
<td>1.0</td>
<td>2.08</td>
<td>0.99</td>
<td>0.761</td>
</tr>
<tr>
<td>2.0</td>
<td>4.16</td>
<td>1.0</td>
<td>1.07</td>
</tr>
<tr>
<td>3.0</td>
<td>6.24</td>
<td>1.02</td>
<td>1.31</td>
</tr>
<tr>
<td>4.0</td>
<td>8.32</td>
<td>1.02</td>
<td>1.52</td>
</tr>
<tr>
<td>5.0</td>
<td>1.04 \times 10^{18}</td>
<td>1.02</td>
<td>1.69</td>
</tr>
</tbody>
</table>

SOLUTION
(b) For a base design with $qN_bW_b = 1.6 \times 10^{-6}$ C/cm$^2$, we have the following table

<table>
<thead>
<tr>
<th>$J_C$ (mA/µm$^2$)</th>
<th>$C_{dBC}$ (fF/µm$^2$)</th>
<th>$V_D$ (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.243</td>
<td>65.8</td>
</tr>
<tr>
<td>0.2</td>
<td>0.343</td>
<td>46.6</td>
</tr>
<tr>
<td>0.3</td>
<td>0.420</td>
<td>38.1</td>
</tr>
<tr>
<td>0.4</td>
<td>0.484</td>
<td>33.1</td>
</tr>
<tr>
<td>0.5</td>
<td>0.540</td>
<td>29.6</td>
</tr>
<tr>
<td>0.8</td>
<td>0.659</td>
<td>24.3</td>
</tr>
<tr>
<td>1.0</td>
<td>0.761</td>
<td>21.0</td>
</tr>
<tr>
<td>2.0</td>
<td>1.07</td>
<td>15.0</td>
</tr>
<tr>
<td>3.0</td>
<td>1.31</td>
<td>12.2</td>
</tr>
<tr>
<td>4.0</td>
<td>1.52</td>
<td>10.5</td>
</tr>
<tr>
<td>5.0</td>
<td>1.69</td>
<td>9.5</td>
</tr>
</tbody>
</table>