1.1 You must first specify the indices of two adjacent planes correctly. Some (but not all) adjacent planes are: (100) - (010), (100) - (001), etc. Take the first adjacent pair:

\[
\arctan(\Theta) = \mathbf{n}_1 \cdot \mathbf{n}_2 = (1, 0, 0) \cdot (0, 1, 0) = 0
\]

which implies that \( \Theta \) is 90\(^0\).

1.2. The inversion matrix is just \(-I\):

\[
T_{\text{inv}} = \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

1.3 The fraction of charged donor traps is:

\[
\eta = \frac{1}{1 + \exp\left[\frac{q}{kT}(\phi_t - \phi_f)\right]} = \frac{1}{1 + \exp\left[\frac{(4-565)}{0.02579}\right]} = .9983
\]

The fraction of charged acceptors (negative acceptors) is given by the same expression. Thus:

\[
\eta = \frac{1}{1 + \exp\left[\frac{(0.005-975)}{0.02579}\right]} = 1
\]

I couldn't get Mathematica to return a number smaller than 1 without bit overflow.

1.4. The built in voltage is 0.95. The zero biased capacitance is given as:

\[
C_{sc} = \frac{A\epsilon_{si}}{x_{sc}}
\]

\[
x_{sc} = \sqrt{\frac{2\epsilon_{si}\phi_{bi}}{qN_a}}
\]

Note: I have ignored the donor doping in equation 5. It’s much greater than the acceptor doping, and the lightly doped side defines the space-charge extent.

1.5 First we have to figure out how many dopant atoms are “in the box. The volume of the box is 10\(^{-15}\). So there are 10\(^{20}\) \times 10\(^{-15}\) = 10\(^5\) atoms in the box. The “uncertainty,”
or variance ($\sigma$), in doping is the square root of this number (316 atoms). Thus the 3 $\sigma$ uncertainty is about 1000 atoms. The percentage uncertainty is $(1000/100000) \times 100\% = 1\%$.

2.0 Our discussion proceeds with reference to figure 1.

![Figure 1: Bias modulation and transport in a Schottky diode.](image)

From this figure we see that the “barrier height” ($\phi_{bo}$) is fixed no matter what the bias. The barrier height determines the current flow from left to right, $J_{LR}$. The current incident on the barrier from the left is, as usual:

$$J_{inc} = en_s v_{th}$$

(6)

Where $n_s$ is the number of electrons in the metal whose velocity vector points toward the barrier and $v_{th}$ is the thermal velocity of the carrier. The current, $J_{LR}$, is just this number times the probability of successfully surmounting the barrier. That is just the probability of finding an electron with energy greater than the conduction band edge. We can get this probability by integrating the Fermi-Dirac function from the band edge up to infinity. As
we are only concerned with the “tail” of the FD distribution, we can make the Boltzmann approximation and say

$$F_{FD} \approx \exp\left[\frac{-q}{kT}(\phi - \phi_{fn})\right]$$  \hspace{1cm} (7)

Integrating this function from the conduction band edge potential up to infinity yields:

$$p = \frac{kT}{q} \exp\left(-\frac{q}{kT}\phi_c\right)$$  \hspace{1cm} (8)
as the probability for surmounting the barrier. Thus:

$$J_{LR} = qn_s v_{th} \left(\frac{kT}{q}\right) \exp\left(-\frac{q}{kT}\phi_c\right)$$  \hspace{1cm} (9)

Of course, we could wave our hands and make up another equation for the current flowing right to left. But we do know that whatever that equation is, it will be equal to equation 9 when there is no bias. Applying bias will raise of lower the barrier seen by carriers incident from the right. Thus, the equation for current flow from right to left is

$$J_{RL} = J_{LR} \exp\left(\frac{q}{kT}V_a\right)$$  \hspace{1cm} (10)

And the net current is the difference between these two currents:

$$J_{tot} = J_{LR} - J_{RL} = J_{LR}(1 - \exp\left(\frac{q}{kT}V_a\right))$$  \hspace{1cm} (11)

To summarize, bias changes the carrier injection barrier for mobile charges incident from the right (from the semiconductor) onto the barrier. The injection barrier determines the “success” probability for a carrier to surmount the barrier. The carrier from the metal into the semiconductor is always the same, and is set by $\phi_{bn}$. Increasing the reverse bias increases the barrier on the semiconductor side, lowering the semiconductor to metal current. Eventually, the only current that flows is that from the metal to the semiconductor, and this is a constant (the reverse saturation current). Forward bias lowers the transport barrier for charges injected into the metal from the semiconductor. This exponentially increases the semiconductor to metal current, swamping the metal to semiconductor current.

Forward bias shrinks the physical extent of the barrier, increasing the diodes capacitance (see equation 4, above). Reverse bias expands the depletion, reducing the capacitance.

3.1a As usual:

$$\phi_{bn} = \frac{kT}{q} \ln\left(\frac{N_d}{n_i}\right) = 0.575V$$  \hspace{1cm} (12)

where $n_i = 1.45 \times 10^{10}$ per cubic centimeter. We can solve for $N_D$, to find the maximum doping would be $6.35 \times 10^{19}$. the same would be true if we doped with acceptors.
3.1b. When the Fermi level approaches the conduction band edge, more donor traps “sing” in the Fermi sea and cannot lose their electrons to the conduction band. The traps discharge and the doping efficiency degrades. As the Fermi level moves to the valence band edge, the acceptor traps start to poke up above the Fermi sea and they loose their electrons (discharging the trap). When this happens, the traps can’t create holes in the valence band by hooking on to valence electrons. Here, again, the doping efficiency degrades.

3.1c. From equation 5, we find \[ x_{sc} = 6.73 \times 10^{-7} \text{cm}. \]

3.1d. Reducing the doping concentration by 50% increases the space charge thickness by \[ \sqrt{2} = 1.414, \] making it \[ x_{sc} = 9.52 \times 10^{-7} \text{cm}. \] It reduces the built in drop by reciprocal \[ 0.259 \times \log(4), \] or by about 0.04V, making it 1.11V.

3.1e. You have to supply an extra 0.04V.

3.1f. \[ 9.53 \times 10^{-7} \text{cm} \]

3.1g. The space charge extends \[ 9.53 \times 10^{-7}/2 = 4.265 \times 10^{-7} \text{cm} \] into the p (or n) regions away from the metallurgical junction. The sheet charge density is this number times the electron charge times either \[ N_a \] or \[ N_d = 1.35 \times 10^{13} \text{ per square centimeter}. \] The electric field is \[ \sigma_E = \frac{1.6 \times 10^{-19} \times 1.35 \times 10^{13}}{10^{-12}} = 2.16 \times 10^6 \text{V/cm}. \]

3.2 The “Shockley equation” governs pretty much all diode behavior:

\[ I_d = I_s (\exp \frac{qV_a}{kT} - 1) \]

The small-signal forward resistance of the diode is given by:

\[ r_d = \left( \frac{\partial I_d}{\partial V_a} \right)^{-1} \]

and so, in forward bias, the exponential term in the Shockley equation is much bigger than 1:

\[ r_d = \left( \frac{qI_d}{kT} \right)^{-1} \]

Thus, for the first case, \( I_d \) is 5.5 mA, and \( r_d = 4.74 \Omega \). At 0.2 volts forward bias, the current is over 12A and \( r_d = 0.002 \Omega \). Big difference, huh!

4. Some things to note. You can’t put the metal on the bare silicon. It would form a Schottky contact to substrate ground. You want to “seal the metal up” with a second insulator deposition (a CVD deposition) to prevent the metal from oxidizing. Here is the base flow and a vertical cross-section:
Figure 2: Process flow and vertical cross-section
Here is a top-down (plane) view:

Figure 3: Top-down (plane view) of resistor