The 2-Wattmeter method re-visited!

This is to illustrate the 2-Wattmeter method and its readings as observed in the lab. The circuit analyzed is the general one shown below. It is assumed that the voltages are balanced and the impedances are also balanced. The loads are of the form \( Z = R + jX \) and the voltages (to neutral) are given by:

\[
\begin{align*}
V_1 &= V \angle 0 \\
V_2 &= V \angle -120 \\
V_3 &= V \angle 120
\end{align*}
\]

Let \( \theta \) be the angle of the impedance \( Z \).

Thus \( I_1 = I \angle -\theta \) where \( I = \frac{V}{|Z|} \).

Similarly, \( I_2 = I \angle (-120 - \theta) \) and \( I_2 = I \angle (120 - \theta) \). When the wattmeters are connected as shown, their readings will be equal to:

\[
P_1 = \text{Re} \left( V_{13} I_1^* \right) \quad \text{and} \quad P_2 = \text{Re} \left( V_{23} I_2^* \right).
\]

A program using Matlab is shown below where the value of the reactance is varied from \(-25\) to \(+25\) Ohm.

```matlab
clear; clf;
v=10;  % voltage to neutral
r=5;   % resistance
X=-25:0.1:25;  % reactance from -25 to +25
zero=zeros(size(X));  % x-axis
R=r*(ones(size(X)));  % constant R
Z=R+j*X;  % Impedance vector
V1=v*(ones(size(X)));  % three balanced voltages
V2=v*(cos(2*pi/3)+j*sin(2*pi/3))*(ones(size(X))); 
V3=v*(cos(-2*pi/3)+j*sin(-2*pi/3))*(ones(size(X)));
I1=V1./Z;  % compute the three current vectors
I2=V2./Z;
I3=V3./Z;
V13=V1-V3;  % voltage on top wattmeter
V23=V2-V3;  % voltage on lower wattmeter
P1=real(V13.*conj(I1));  % reading of top wattmeter
P2=real(V23.*conj(I2));  % reading of lower wattmeter
P=P1+P2;  % the three phase power using 2 wattmeters
Pp=3*real(V1.*conj(I1));  % computed three phase power
plot(X,P1,X,P2,X,P,X,zero),grid
xlabel('X in Ohm'),ylabel('Power in Watt'),
text(-20,0,'capacitive X'),text(15,0,'inductive X')
```
The powers plotted are the reading of P1 in blue, the reading of P2 in green and the sum in red. Remarks: The powers of the wattmeters are equal only when \( X = 0 \) and are unequal for all other values! Note also that for some values of \( X \) one of the two wattmeters can change the sign of its reading (this happens when \( \theta = \pm 30^\circ \)). The sum of the two wattmeter readings is still exactly equal to the three phase power. The two wattmeters would read exactly alike if the common voltage connections is to the neutral point, in which case each meter would read only one third of the total three-phase power.