**Notes #1: Power Measurements**

Single phase power equations:

\[ S = VI^* \]

Where \( S \) is the complex power, \( V \) the voltage phasor and \( I \) the current phasor. Note that \( I^* \) is the complex conjugate of the current phasor.

\[ S = P + jQ \]

\( S \) is the complex power into the circuit shown below. The real part of \( S \) is the average power \( P \). The imaginary part \( Q \) is the magnitude of the double frequency \((2\omega)\) component whose average is zero (the average is zero, but the magnitude is \( Q \)). Note that inductors produce positive values of \( Q \) while capacitors produce negative values (i.e. capacitors actually generate \( Q \)). The unit of \( S, P \) and \( Q \) is the watt. However, due to their different nature, each is usually given another unit: \( S \) is usually in VA (volt-ampere), \( P \) is in Watt and \( Q \) is in var (volt-ampere reactive). Of course, we also have the units kVA, MVA, kvar, Mvar, kW, and MW. \( P \) results only from the presence of resistance in the circuit. In a passive system, \( P \) is always positive, while \( Q \) can be positive or negative (inductive or capacitive) or zero.

\[ P = V_{rms} I_{rms} \cos(\theta) \quad \text{and} \quad Q = V_{rms} I_{rms} \sin(\theta) \]

\( PF = \cos(\theta) \) lagging or leading (inductive or capacitive). \( \theta \) is the angle by which the current \( I \) leads the voltage \( V \). \( \theta \) is negative if the current lags the voltage.

\[ \theta = \angle S = \tan^{-1}(Q / P) = \angle Z \]

Where \( Z \) is the impedance of the network as shown.

\[ Z = R + jX \]

\( R \) is the resistance and \( X \) the reactance.
Three phase power equations:

We assume a balanced three phase system and observe the lines that go to the load on the right as shown in the figure next page. Without knowing the value of the load, and not knowing the kind of connection, the following equations are always true for a balanced system.

\[ S_{3p} = \sqrt{3} V I^* \]  This is the total three phase complex power to the system shown below. It is assumed that the supply voltage \( V = V_{L-L} \) is a balanced three phase voltage from line to line (the phase voltage would be \( V_{ph} = V / \sqrt{3} \)). As usual, the asterisk denotes complex conjugation. \( I \) is the line current.

\[ P = \sqrt{3} V_{rms} I_{rms} \cos(\theta) \] and \[ Q = \sqrt{3} V_{rms} I_{rms} \sin(\theta). \] \( \theta \) is defined as the angle by which the current \( I \) leads the voltage \( V \). For an inductive circuit, \( \theta \) is negative, while it is positive for a capacitive circuit. [Memory aid: ELI and ICE].

\[ PF = \cos(\theta) \] The cosine of the angle \( \theta \) is called the power factor. Since the sign of \( \theta \) is lost after the cosine function, we often use the words lagging or leading in order to determine the sign of \( \theta \). The triangle below shows the relations between \( P, Q, S, \) and \( \theta \) for an inductive circuit (lagging power factor). A capacitive circuit would have negative \( Q \) and a leading \( PF \).
Remarks: In a three phase system, using two wattmeter as shown in experiment 1, we have (applying KCL):

\[ \sum_{k=1}^{3} i_k = 0. \]  
Here we let \( i_a = i_1, \ i_b = i_2, \ i_c = i_3. \) Note that the voltage on the top wattmeter is \((v_c - v_b),\) and the voltage on the bottom wattmeter is \((v_a - v_b).\) Thus the total reading of the two wattmeters would be:

\[ p = p_1 + p_2 = i_c(v_c - v_b) + i_b(v_a - v_b). \]  
When this is expanded we have: \( p = i_c v_c + i_a v_a - v_b(i_a + i_c). \) But we also know that \((i_a + i_c) = -i_b,\) hence the expression for \( p \) becomes: \( p = i_c v_c + i_a v_a + i_b v_b, \) which is the total power (assuming no neutral connection). Thus the sum of the reading of the two wattmeters is equal to the total power. This is true regardless of the type of load (balanced or not). While the expression for power just found is the instantaneous power, this would give the wattmeter readings if we take the average of the expression. Thus, \( P = P_1 + P_2 \) would give the total average power.

Clearly, if the load is resistive, then we would always have \( P_1 = P_2, \) and only one wattmeter is needed. It is noted that the total three phase power would be thus twice the reading of the one wattmeter. However, if the load is reactive (RL or RC or RLC) then the two meters would not read the same even if the load is balanced!

How do we know that \( p = i_c v_c + i_a v_a + i_b v_b, \) is indeed the total power? Here the voltages are the line voltages with respect to some arbitrary reference potential, say \( v_0. \) Thus the exact power equation is \( p = i_c (v_c - v_0) + i_a (v_a - v_0) + i_b (v_b - v_0). \) Since there is no neutral connection, then \( i_a + i_b + i_c = 0 \) and this last equation reduces to \( p = i_c v_c + i_a v_a + i_b v_b. \) Assuming the neutral is \( n, \) then the three voltages may be taken with respect to this neutral (i.e. \( v_a = v_{an} = v_a - v_n, \) etc. . .). Thus \( p = p_c + p_a + p_b \) which is the total power. Note that \( v_a - v_b \) is line to line voltage and is larger than the phase voltage: \( |v_a - v_b|_{rms} = \sqrt{3} \ |v_a|_{rms} \) (for balanced three phase systems.)