Problem Set 3
Fall 2003

Issued: Monday, September 22, 2003
Due: Wednesday, October 1, 2003

Reading Assignment: Oppenheim & Schafer: Chapter 4, Sections 4.0–4.6.

Problem 3.1

Oppenheim & Schafer, problem 4.22.

Problem 3.2

Oppenheim & Schafer, problem 4.25.

Problem 3.3

Oppenheim & Schafer, problem 4.28.

Problem 3.4

Oppenheim & Schafer, problem 4.33.

Problem 3.5

Oppenheim & Schafer, problem 4.34.
Problem 3.6

Consider a continuous-time signal $x_c(t)$ with a Fourier transform $X_c(j\Omega)$ that is zero except for $\frac{2}{3}\Omega_o < |\Omega| < \Omega_o$. For sketching purposes, throughout the problem you may assume that $x_c(t)$ has the Fourier transform shown in Figure 3.6-1; however, keep in mind that the systems you construct should work for signals with arbitrary $X_c(j\Omega)$ for $\frac{2}{3}\Omega_o < |\Omega| < \Omega_o$.

(a) A continuous-time signal $x_r(t)$ is obtained through the process shown in Figure 3.6-2. First $x_c(t)$ is sampled with sampling period $T_1$ to obtain $x_1[n]$ and then $x_1[n]$ is converted to $x_s(t)$ through a sample to impulse (S/I) converter with sampling period $T_1$. An S/I converter converts the input discrete-time sequence into a continuous-time impulse train, i.e.,

$$x_s(t) = \sum_{n=-\infty}^{+\infty} x[n] \delta(t - nT_1).$$

Finally, $x_s(t)$ is passed through a low pass filter with frequency response $H_r(j\Omega)$. $H_r(j\Omega)$ is shown in Figure 3.6-3.

Determine the range of values for $T_1$ for which $x_r(t) = x_c(t)$.

(b) Consider the system in Figure 3.6-4. The S/I system in this case is the same as the one in part (a), except that the sampling period is now $T_2$. The system $H_s(j\Omega)$ is some continuous-time ideal LTI filter. We want $x_o(t)$ to be equal to $x_c(t)$ for all $t$, i.e., $x_o(t) = x_c(t)$ for some choice of $H_s(j\Omega)$. Find all values of $T_2$ for which $x_o(t) = x_c(t)$ is possible.

Figure 3.6-1 Fourier transform $X_c(j\Omega)$

Figure 3.6-2 Conversion system for part (a)
Figure 3.6-3 Frequency response $H_r(j\Omega)$

For the largest $T_2$ you determined which would still allow recovery of $x_c(t)$, choose $H_s(j\Omega)$ so that $x_o(t) = x_c(t)$. Sketch $H_s(j\Omega)$.

**Problem 3.7 (optional)**

*Oppenheim & Schafer*, problem 4.45.