1. \[ w_e = 10^8, \ k_f = 10^5, \ k_p = 25. \]

\[ \varphi_{FM}(t) = A e^{\cos(w_e t + k_f \int m(t) dt + \theta_0)} \]  
\[ \varphi_{PM}(t) = A e^{\cos(w_e t + k_p m(t) + \theta_0)} \]

We pick \( A = 1 \) and \( \theta_0 = 0 \).

For \( \varphi_{FM}(t) \), the instantaneous frequency \( \omega_i = 10^8 + 10^5 m(t) \) (rad/s).

Since \( m(t) \in [-1, 1] \), \( \omega_i \in [10^8 - 10^5, 10^8 + 10^5] \cong [9.99 \times 10^7, 1.001 \times 10^8] \) (rad/s).

For \( \varphi_{PM}(t) \), \( \omega_i = 10^8 + 25 \frac{dm(t)}{dt} \). Since \( \frac{dm(t)}{dt} \in [-8000, 8000] \), we have \( \omega_i \in [10^8 - 2 \times 10^5, 10^8 + 2 \times 10^5] \cong [9.98 \times 10^7, 1.002 \times 10^8] \) (rad/s).

* Here \( k_f \) and \( k_p \) follow definitions in the textbook.

\( (k_f = 2\pi k_p) \)
2. 5.1-2 \( \omega_c = 2 \pi \times 10^6 \), \( k_f = 2000 \pi \), \( k_p = \frac{\pi}{2} \)

(a) See sketches below. For \( \varphi_{pm}(t) \), instantaneous frequency \( f_i = 10^6 + 1000m(t) \) Hz
\( \Rightarrow f_i \in [999k, 1001k] \) Hz.
See (b) for \( \varphi_{pm}(t) \) case.

(b) Without loss of generality, we assume one cycle of the sawtooth of \( m(t) \) is centered at the origin. Over that cycle, \( m(t) = 2000t \) and
\( \varphi_{pm}(t) = A \cos(2 \pi \times 10^6 t + \frac{\pi}{2} \times 2000 t) = A \cos(2 \pi (10^6 + 500)t) \).

At discontinuities of \( m(t) \), i.e., \( t = \frac{T}{2}, \frac{3}{2}T, \frac{5}{2}T, \ldots \), there is a phase jump from \( \frac{\pi}{2} \) to \( -\frac{\pi}{2} \), which means there is a phase shift \(-\pi\) at these
\( \varphi(t) \) discontinuities. Hence, \( \varphi_{pm}(t) = A \cos(2 \pi (10^6 + 500)t + \varphi(t)) \)
\( \Rightarrow \varphi_{pm}(t) \) is equivalent to another PM signal with
\( f_c = 10^6 + 500 \) Hz modulated by \( g(t) \)

Furthermore, we need to have \( k_p \Delta < 2\pi \) to avoid phase ambiguity, where \( \Delta = +1 - (-1) = 2 \)
Therefore, \( k_p \) has to be less than \( \frac{2 \pi}{\Delta} = \pi \)
3. \[ w_c = 2\pi \times 10^3 \text{ rad/s} \]

(a) \[ k_p = 20\pi \]
\[ S_{FM}(t) = A e^{\cos \left( w_c t + k_p \int_{-\infty}^{t} m(t) \, dt + \theta_0 \right)} \]
with \( A = 1 \), \( \theta_0 = 0 \)
\[ f = 10^3 + 10 m(t) \]
\( f \in [10^3 - 10 \times 3, 10^3 + 10 \times 3] = [970, 1030] \text{ Hz} \)

(b) \[ k_p = \frac{\pi}{2} \]
\[ S_{PM}(t) = A e^{\cos \left( w_c t + k_p m(t) + \theta_0 \right)} \]
with \( A = 1 \), \( \theta_0 = 0 \)
\[ f = 10^3 + \frac{1}{4} \frac{d m(t)}{dt} \]
\( \frac{d m(t)}{dt} \in [-60, 60] \Rightarrow f \in [10^3 - \frac{60}{4}, 10^3 + \frac{60}{4}] = [985, 1015] \text{ Hz} \)

4. \[ w_c = 10000\pi \]
\[ \varphi_{EM}(t) = 10 \cos (13000\pi t) \text{, } |t| \leq 1 \]

(a) \[ k_p = 1000 \text{, } w_c t + k_p m(t) = 13000\pi t \Rightarrow m(t) = \frac{1}{k_p} [13000\pi t - w_c t] = 3\pi t \text{, } |t| \leq 1 \]

(b) \[ k_f = 1000 \text{, } w_c t + k_f \int_{-\infty}^{t} m(t) \, dt = 13000\pi t \Rightarrow \int_{-\infty}^{t} m(t) \, dt = \frac{1}{k_f} [13000\pi t - w_c t] \]
\[ = 3\pi t \text{, } |t| \leq 1 \]
\[ \Rightarrow m(t) = \frac{d(3\pi t)}{dt} = 3\pi \text{, } |t| \leq 1 \]
(a) $k_f = 500 \pi$

$\phi_{FM}(t) = A_c \cos \left( w_c t + k_f \int_{0}^{t} m(t) dt + \Theta_0 \right)$, with $A_c = 1$, $\Theta_0 = 0$.

$f_c = 2000 + 250 m(t) \Rightarrow f_c \in [2000 - 250 \times 4, 2000 + 250 \times 4] = [1600, 3000]$ Hz

(b) $k_p = 0.25 \pi$

$\phi_{PM}(t) = A_c \cos \left( w_c t + k_p m(t) + \Theta_0 \right)$, with $A_c = 1$, $\Theta_0 = 0$.

$f_i = 2000 + \frac{1}{8} \frac{d|m(t)|}{dt}$, since $\frac{d|m(t)|}{dt} \in [-400, 800]$, we have

$f_i \in [2000 - \frac{1}{8} \times 400, 2000 + \frac{1}{8} \times 800] = [1950, 2100]$ Hz

\[5.1 - 5\]

$w_c = 4 \pi \times 10^3$ rad/s
6. \[ 5.2-1 \]
For Prob. 5.1-3, bandwidth of \( m(t) \) is approximated by its own 5th-harmonic frequency. Hence \( B = \frac{5}{T} = \frac{5}{0.25} = 20 \text{ Hz} \).

(a) For FM signal, \( B_{FM} = 2(\Delta f + B) \), \( \Delta f = \frac{k_{P} m_{p}}{2\pi} = \frac{20\pi \times 3}{2\pi} = 30 \text{ Hz} \) \( (5.13) \)
\[ \Rightarrow B_{FM} = 2(30 + 20) = 100 \text{ Hz} \]

(b) For PM signal, \( B_{PM} = 2(\Delta f + B) \), \( \Delta f = \frac{k_{P} m_{p}}{2\pi} = \frac{15\pi \times 60}{2\pi} = 15 \text{ Hz} \) \( (5.18) \)
\[ \Rightarrow B_{PM} = 2(15 + 20) = 70 \text{ Hz} \]

7. \[ 5.2-2 \]
For Prob. 5.1-5, assume bandwidth of \( m(t) \) is its 5th-harmonic frequency.
That is, \( B = \frac{5}{T} = \frac{5}{0.04} = 125 \text{ Hz} \)

(a) For FM signal, \( B_{FM} = 2(\Delta f + B) \), \( \Delta f = \frac{k_{P} m_{p}}{2\pi} = \frac{500\pi \times 4}{2\pi} = 1000 \text{ Hz} \) \( (5.13) \)
\[ \Rightarrow B_{FM} = 2(1000 + 125) = 2250 \text{ Hz} \]

(b) For PM signal, \( B_{PM} = 2(\Delta f + B) \), \( \Delta f = \frac{k_{P}[\hat{m}(t)_{\text{max}} - \hat{m}(t)_{\text{min}}]}{2x2\pi} = \frac{\pi}{4}[800 - (-400)] \]
\[ = \frac{1200}{4\pi} = 75 \text{ Hz} \]
\[ \Rightarrow B_{PM} = 2(75 + 125) = 400 \text{ Hz} \]

8. \[ 5.2-4 \]
\( \varphi_{EM}(t) = 10 \cos(\omega_{c}t + 0.1 \sin(2000\pi t)) \), \( \omega_{c} = 2\pi \times 10^{6} \)

(a) \( p = \frac{1}{2} \times 10^{2} = 50 \) \( \text{ (P.256 of textbook) } \)

(b) \( \theta(t) = \omega_{c}t + 0.1 \sin(2000\pi t) \Rightarrow \omega(t) = \frac{d\theta(t)}{dt} = \omega_{c} + 200\pi \cos(2000\pi t) \)
\[ \Rightarrow \Delta f = \frac{200\pi}{2\pi} = 100 \text{ Hz} \] \( \text{ (frequency deviation) } \)

(c) \( \theta(t) = \omega_{c}t + 0.1 \sin(2000\pi t) \)
\[ \Rightarrow \text{ phase deviation } \Delta \phi = 0.1 \text{ rad} \]

(d) \( B = \frac{200\pi}{2\pi} = 1000 \text{ Hz} \)
\[ \Rightarrow B_{EM} = 2(\Delta f + B) = 2(100 + 1000) = 2200 \text{ Hz} \]

\[ * \]

P. 5
9. \[ J_k(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{i(\beta \sin x - kx)} \, dx \quad k = 0, \pm 1, \pm 2, \ldots, \beta \in \mathbb{R} \]

\[ = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left[ \cos(\beta \sin x - kx) + i \sin(\beta \sin x - kx) \right] \, dx \]

\[ = \frac{1}{2\pi} \int_{-\pi}^{\pi} \cos(\beta \sin x - kx) \, dx + \frac{1}{2\pi} \int_{-\pi}^{\pi} \sin(\beta \sin x - kx) \, dx \]

We note \[ \cos(\beta \sin(-x) - k(-x)) = \cos(-\beta \sin x + kx) = \cos(\beta \sin x - kx) \]

\[ \sin(\beta \sin(-x) - k(-x)) = \sin(-\beta \sin x + kx) = -\sin(\beta \sin x - kx) \]

Thus, \( \cos(\beta \sin x - kx) \) is an even function of \( x \)

\( \sin(\beta \sin x - kx) \) is an odd function of \( x \)

\[ \Rightarrow J_k(\beta) = \frac{1}{2\pi} \times 2 \int_{0}^{\pi} \cos(\beta \sin x - kx) \, dx + \frac{1}{2\pi} \times 1 \times 0 \]

\[ = \frac{1}{\pi} \int_{0}^{\pi} \cos(\beta \sin x - kx) \, dx \in \mathbb{R} \]

Therefore, \( J_k(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{i(\beta \sin x - kx)} \, dx \) is real, for \( \beta \in \mathbb{R} \), \( k = 0, \pm 1, \ldots \).
10. Take USB modulation/demodulation for example.

Note: $f_0$ is determined by Group number and $f_s$ is determined by Super-group number.
10. (Continued)

Also refer to figures shown below [1]:

- Figure 1: Block diagram of DSB FDM transmission system

- Figure 2(a): Block diagram of SSB-SC FDM long-haul transmission system

- Figure 2(b): The corresponding frequency plan for 12-channel SSB group

REFERENCE