ENEE 380 Solutions to Exam I Spring 2009 Prof. E. Ott

Prob. 1 (a) $V=\int \frac{\overset{\rho_\ell\, d\ell}{dq}}{R}\frac{1}{4\pi\epsilon_0}$, $R=\sqrt{a^2+z^2}$, $V(z)=\frac{1}{4\pi\epsilon_0\sqrt{a^2+z^2}}\overbrace{\int\rho_\ell\, d\ell}^{2\pi a\rho_\ell}=\frac{2\pi a\rho_\ell}{4\pi\epsilon_0\sqrt{a^2+z^2}}=\frac{a\rho_\ell}{2\epsilon_0\sqrt{a^2+z^2}}$ (25pts)

(b) External work $= q\left[V(z=0)-V(z=a)\right]=\frac{a\rho_\ell q}{2\epsilon_0}\left(\frac{1}{a}-\frac{1}{a\sqrt{2}}\right)=0.147\,\rho_\ell q/\epsilon_0$ (13 pts.)

Prob. 2 $\oint_S \underline{D}\cdot d\underline{s}=Q$, $S=$ sphere of radius $R \Rightarrow 4\pi R^2 D_R=Q$

$\Rightarrow \underline{D}=D_R\underline{a}_R=(Q/4\pi R^2)\underline{a}_R$ for both $R<a$ and $R>a$.

$$\underline{E}=\underline{D}/\epsilon=\begin{cases} Q/4\pi\epsilon_0R^2 \text{ in } R<a \text{ (where } \epsilon=\epsilon_0), \\ Q/\underbrace{4\times 3}_{12}\pi\epsilon_0R^2 \text{ in } R>a \text{ (where } \epsilon=3\epsilon_0). \end{cases}$$

(20 pts.)

Prob. 3 (a) $\nabla\times\underline{E}=\underline{a}_z\,\partial/\partial x(3x)=3\underline{a}_z\neq 0$. But $\nabla\times\underline{E}=0$ in electrostatics.

$\Rightarrow$ this cannot be an electrostatic electric field. (10 pts.)

(b) See p. 102 of textbook for derivation. $E_x=-\rho_s/\epsilon_0$. (20 pts.)

(c) $\underline{E}=-\nabla V=-4xy\,\underline{a}_x-2x^2\underline{a}_y$. At $(2,1,1)$: $\underline{E}=-8\underline{a}_x-8\underline{a}_y$. (12 pts.)

$\underline{F}=\overset{-2}{Q}\underline{E}=16\underline{a}_x+16\underline{a}_y$.

Exam Statistics

Median grade = 61