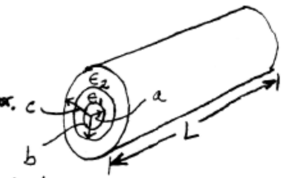
ENEE 380
PROF. E. OTT

FINAL EXAM

FALL 2000

(Total points: 160)



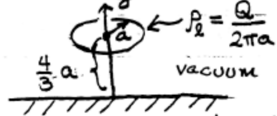
Problem 1 (25 points) Consider a long coaxial capacitor. The space between the conductors is filled with a dielectric of dielectric constant $\epsilon_1 = 2\times10^{-11}$ Farads/m. between $r = a = 10^{-2}$ m. and $r = b = 2\times10^{-2}$ m. and with dielectric constant $\epsilon_2 = 4\times10^{-11}$ Farads/m. between $r = 2\times10^{-2}$ m. and $r = 3\times10^{-2}$ m. Find the capacitance if the length of the capacitor is $L = 0.2$ m. (Neglect fringing.)

Problem 2 (30 points) If the potential is given by $V(x,y,z) = 3\sin\pi x$ Volts,

(a) what is the electric field $\underline{E}(x,y,z)$?

(b) what is the volume charge density $\rho(x,y,z)$ if $\epsilon = \epsilon_0$?

(c) how much work does one have to do to move a charge of -0.3 coulombs from the point $(x=4.25, y=0, z=0)$ to the point $(x=0, y=2.00, z=0)$?



Problem 3 (25 points) A circular line charge of density $\rho_\ell = Q/(2\pi a)$ $\frac{\text{Coulombs}}{\text{meter}}$ and radius a is situated in the plane $z = \frac{4}{3}a$ in vacuum. The region $z<0$ is occupied by a grounded perfect conductor. What is the force on a point charge of $-3Q$ coulombs situated at the center of the circle? Express your answer in terms of Q, a and ϵ_0.

Problem 4 (20 points) Given a current density $\underline{J} = (5t\cos\pi x)\underline{a}_x$ Amperes/meter^2, what is the total charge $Q(t)$ in the cube $0\le x\le 1/2$ meter, $0\le y\le \frac{1}{2}$ meter, $0\le z\le\frac{1}{2}$ meter, if the total charge in the cube is zero at $t=0$?

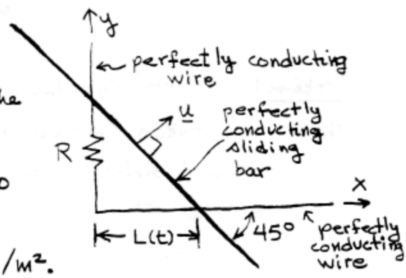
Problem 5 (20 points) Given a current density

$$\underline{J} = \begin{cases} 3\underline{a}_z & \text{for } r<a \\ 0 & \text{for } r>a, \end{cases}$$ (where r is in cylindrical coordinates),

use Ampere's law to find $\underline{H}$ in $r<a$ and in $r>a$.

Problem 6

(40 points) As shown, a perfectly conducting sliding bar in electral contact with wires on the positive x-axis and the positive y-axis is moving with a constant speed $u = 5$ m./s. so that $L(t) = 5\sqrt{2}\,t$ meters. There is a uniform magnetic field $\underline{B} = 2\,\underline{a}_z$ Webers/m^2.



At the instant that $L(t) = 1$ meter the external force exerted on the bar necessary to keep it moving at $u = 5$ m./s. is $F = 5$ Newtons.

(a) What is the magnetic flux $\Phi(t)$ through the triangular region bounded by the bar and the x and y axes?

(b) What is the voltage across the resistor at the instant of time when $L(t) = 1$ meter?

(c) What is the current when $L(t) = 1$ meter?

(d) What is the resistance of the resistor?

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Outline of Solution to Fall 2000 Final

Problem 1 $\underline{D} = D_r(r)\,\underline{a}_R \qquad 2\pi r L D_r(r) = Q \qquad D_r = \frac{Q}{2\pi r L}$

$$E_r = D_r/\epsilon = \begin{cases} Q/(2\pi\epsilon_1 r L) & \text{in } b > r > a \\ Q/(2\pi\epsilon_2 r L) & \text{in } c > r > b \end{cases}$$

$$V = \int_a^b E_r\,dr + \int_b^c E_r\,dr = \frac{Q}{2\pi L}\left[\frac{1}{\epsilon_1}\ln\frac{b}{a} + \frac{1}{\epsilon_2}\ln\frac{c}{b}\right] = \frac{Q}{C}$$

$$C = \left(\frac{1}{2\pi L}\left[\frac{1}{\epsilon_1}\ln\frac{b}{a} + \frac{1}{\epsilon_2}\ln\frac{c}{b}\right]\right)^{-1}$$

Problem 2 (a) $\underline{E} = -\nabla V = -3\pi\ \cancel{\ }\cos\pi x\ \underline{a}_x$

(b) $\rho = \epsilon_0\,\nabla\cdot\underline{E} = \epsilon_0 3\pi^2 \sin\pi x$

(c) $\text{Work} = -0.3 \times [V(\cancel{4.25}\ 0, 2, 0) - V(4.25, 0, 0)]$

$= -0.3 \times \cancel{\ }[0 - 3\sin(4\pi + \frac{\pi}{4})] = 0.9\sqrt{2}$

Problem 3 $\underline{F}$ = force on $-3Q$ charge $= \underline{F}_1 + \underline{F}_2 + \underline{F}_3$

$\underline{F}_1$ = force due to $Q/2\pi a$ line charge density at $z = \frac{4}{3}a$

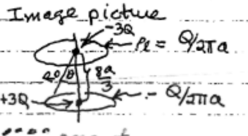
$= 0$ due to symmetry

$\underline{F}_2$ = force due to image charge $+3Q$ at $z = -\frac{4}{3}a$

$= -\underline{a}_z\,(3Q)^2/[4\pi\epsilon_0(8/3)^2] = \ldots$

$\underline{F}_3$ = force due to line charge density $-Q/2\pi a$ at $z = -4/3\,a$

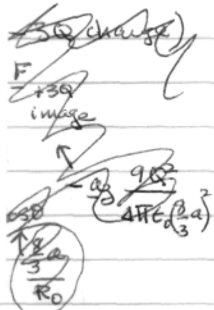
$= -3Q\,E_z(\text{due to } -\frac{Q}{2\pi a})\,\underline{a}_z$ Image picture



$= -\frac{3Q}{4\pi\epsilon_0}\int \frac{\cos\theta}{R_0^2}\left(\frac{-Q}{2\pi a}\right) dl\,\underline{a}_z$

$= \frac{3Q^2\cos\theta}{4\pi\epsilon_0 R_0^2}\underline{a}_z \quad R_0^2 = a^2 + \left(\frac{8a}{3}\right)^2$

$\cos\theta = (8a/3)/R_0$



Problem 4

area of $x=0$ & $x = 1/2$ faces

$$\frac{dQ}{dt} = -\oint \underline{J}\cdot d\underline{s} = -\left(\tfrac{1}{2}\times\tfrac{1}{2}\right)(-5t + 0)$$

$\underline{J}\cdot\underline{a}_n$ on $x=\frac{1}{2}$ face ($\cos\frac{\pi}{2} = 0$)

$\underline{J}\cdot\underline{a}_n$ on $x=0$ face ($\cos 0 = 1$)

$$\frac{dQ}{dt} = +\tfrac{5}{4}t \qquad Q(t) = \tfrac{5}{4}t^2/2$$

Problem 5

$$\underbrace{\oint \underline{H}\cdot d\underline{l}}_{2\pi r H_\phi(r)} = I_s = \begin{cases} 3\pi r^2 & \text{for } r < a \\ 3\pi a^2 & \text{for } r > a \end{cases}$$

$$\underline{H} = \underline{a}_\phi H_\phi(r) = \frac{\underline{a}_\phi}{2\pi r}\begin{cases} 3\pi r^2 & \text{for } r<a \\ 3\pi a^2 & \text{for } r>a \end{cases} = \underline{a}_\phi\begin{cases} \frac{3}{2}r & \text{for } r<a \\ \frac{3}{2}\frac{a^2}{r} & \text{for } r>a \end{cases}$$

Problem 6 Solve using the following Eqs.: $A = \text{Area} = \frac{1}{2}L^2(t)$

$\Phi = BA(t) = \frac{1}{2}BL^2(t)$, Force $= BIL$, Voltage $= IR = d\Phi/dt$