ENEE 322: Signals and Systems
Test 01
October 07, 2004

UNIVERSITY OF MARYLAND, COLLEGE PARK

Rules:

- This exam is CLOSED book and CLOSED notes. No calculator is permitted.
- A page of potentially helpful formulas are attached.
- Perform all work on the provided paper. You may write on the backside of the pages.
- Clearly mark your final answers.
- You must show all of your work and explain your answers to obtain full credit.
- Your answers must be legible and organized; disorganized answers will be penalized.

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Name: ________________

Solution

Student ID: ________________

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1. (10 pts) For the signal shown below, determine an expression for its odd component, and sketch the odd component.

\[ x_o(t) = \frac{1}{2} [x(t) - x(-t)] \]

\[ x(t) = \begin{cases} 
  t + 2 & -2 < t < -1 \\
  1 & -1 < t < 2 \\
  -t + 3 & 2 < t < 3 
\end{cases} \]

\[ x(-t) = \begin{cases} 
  -t + 2 & -2 < -t < -1 \\
  1 & -1 < -t < 2 \\
  t + 3 & 2 < -t < 3 
\end{cases} \]

\[ x_o(t) = \begin{cases} 
  \left[0 - (t + 3)\right]/2 & -3 < t < -2 \\
  \left[(t + 2) - 1\right]/2 & -2 < t < -1 \\
  \left[(1 - 1)\right]/2 & -1 < t < 1 \\
  \left[1 - (-t + 2)\right]/2 & 1 < t < 2 \\
  \left[(-t + 3) - 0\right]/2 & 2 < t < 3 
\end{cases} \]
2. (10 pts) Consider a continuous-time system with impulse response,

\[ h(t) = e^{-2t}u(t + 1). \]

Determine the output signal, \( y(t) \), when the input signal is

\[ x(t) = u(t + 2) - u(t - 2). \]

Recall the convolution integral

\[ y(t) = \int_{-\infty}^{\infty} x(t - \tau)h(\tau)d\tau \]

Draw some pictures to determine regions

When \( t + 2 < -1 \) there is no overlap, so

\[ y(t) = 0 \quad t < -3 \]

When \( t - 2 < -1 < t + 2 \) we get overlap from \( \tau = -1 \) to \( \tau = t + 2 \), so

\[ y(t) = \int_{-1}^{t+2} e^{-2\tau}d\tau = -\frac{1}{2} e^{-2(t+2)} + \frac{1}{2} e^{-2(-1)} = \frac{1}{2} \left( e^2 - e^{-2(t+2)} \right) \quad -3 < t < 1 \]

When \( -1 < t - 2 \) we get overlap from \( \tau = t - 2 \) to \( \tau = t + 2 \), so

\[ y(t) = \int_{t-2}^{t+2} e^{-2\tau}d\tau = -\frac{1}{2} e^{-2(t+2)} + \frac{1}{2} e^{-2(t-2)} = \frac{1}{2} \left( e^{-2(t-2)} - e^{-2(t+2)} \right) \quad 1 < t \]

Hence,

\[
 y(t) = \begin{cases} 
 0 & t < -3 \\
 \frac{1}{2} \left( e^2 - e^{-2(t+2)} \right) & -3 < t < 1 \\
 \frac{1}{2} \left( e^{-2(t-2)} - e^{-2(t+2)} \right) & 1 < t 
\end{cases}
\]
3. (10 pts) Determine whether or not the following signal is periodic. If it is periodic, determine the fundamental frequency, $\omega_0$, and the coefficients in the complex exponential Fourier series.

$$x(t) = \cos(5\pi t)\sin(10\pi t) + \cos(20\pi t)$$

$$x(t) = \cos(5\pi t)\sin(10\pi t) + \cos(20\pi t)
= \left(\frac{e^{j5\pi t} + e^{-j5\pi t}}{2}\right)\left(\frac{e^{j10\pi t} - e^{-j10\pi t}}{j2}\right) + \left(\frac{e^{j20\pi t} + e^{-j20\pi t}}{2}\right)
= \frac{1}{j4} \left[ e^{j15\pi t} - e^{-j5\pi t} + e^{j5\pi t} - e^{-j15\pi t} \right] + \frac{1}{2} \left[ e^{j20\pi t} + e^{-j20\pi t} \right]
= \frac{1}{j4} e^{j(5\pi)(3)t} - \frac{1}{j4} e^{j(5\pi)(-1)t} + \frac{1}{j4} e^{j(5\pi)(1)t} - \frac{1}{j4} e^{j(5\pi)(-3)t} + \frac{1}{2} e^{j(5\pi)(4)t} + \frac{1}{2} e^{j(5\pi)(-4)t}
$$

Hence, the fundamental frequency and period are

$$\omega_0 = 5\pi \quad T = \frac{2\pi}{\omega_0} = \frac{2\pi}{5\pi} = \frac{2}{5}$$

The Fourier series coefficients are

$$a_{-4} = \frac{1}{2}, \quad a_{-3} = -\frac{1}{j4}, \quad a_{-1} = -\frac{1}{j4}, \quad a_1 = \frac{1}{j4}, \quad a_3 = \frac{1}{j4}, \quad a_4 = \frac{1}{2}$$

and $a_n = 0$ for all other $n$. 

4. (10 pts) A discrete-time LTI system has impulse response \( h[n] \) which is \( N \)-periodic, i.e. \( h[n] = h[n + N] \) for all \( n \). Furthermore, it is known that \( h[n] \leq M < \infty \) for all \( n \), and \( h[0] = c \neq 0 \).

(a) (3 pts) Is the system causal? Explain your answer.

(b) (7 pts) Is the system stable? Explain your answer. If the system is unstable, what restrictions, if any, on the input signal \( x[n] \) will guarantee that the output signal is bounded.

(a) For an LTI system to be causal the impulse response must satisfy
\[
h[n] = 0 \quad \forall \quad n < 0.
\]
The current system has an \( N \)-periodic impulse response with
\[
h[-N] = h[0] = c \neq 0
\]
Hence, the system is noncausal.

(b) For an LTI system to be stable, the impulse response must be absolutely summable.
\[
\sum_{n=-\infty}^{\infty} |h[n]| \geq \sum_{n=0, \pm N, \pm 2N, \ldots} |h[n]| = \sum_{n=0, \pm N, \pm 2N, \ldots} |h[0]| = \sum_{n=0, \pm N, \pm 2N, \ldots} c = \infty
\]
Hence, the system is NOT stable.
To find restrictions on the input, recall that
\[
y[n] = x[n] * h[n] = h[n] * x[n]
\]
so we can view this as passing a signal \( h[n] \) through and LTI system with impulse response \( x[n] \). Since \( h[n] \) is bounded, the output will be bounded as long as \( x[n] \) is absolutely summable
\[
\sum_{n=-\infty}^{\infty} |x[n]| < \infty
\]
5. (10 pts) A discrete-time system is described by the input-output relation

\[ y[n] = \mathcal{E} \{ x[n] \} \]

i.e., the output is the even part of the input.

(a) (5 pts) Determine whether or not the system time invariant.

(b) (5 pts) Determine whether or not the system is invertible. If it is invertible, determine the inverse system.

(a) (?) pts Consider passing an arbitrary signal, \( x_1[n] \), through the system,

\[ y_1[n] = \mathcal{E} \{ x_1[n] \} = \frac{x_1[n] + x_1[-n]}{2} \]

and delay the output

\[ y_1[n - n_0] = \frac{x_1[n - n_0] + x_1[-(n - n_0)]}{2} = \frac{x_1[n - n_0] + x_1[-n + n_0]}{2} \]

Now consider a delayed input \( x_2[n] = x_1[n - n_0] \)

\[
\begin{align*}
x_2[-n] &= x_1[-n - n_0] \\
y_2[n] &= \mathcal{E} \{ x_2[n] \} = \frac{x_2[n] + x_2[-n]}{2} = \frac{x_1[n - n_0] + x_1[-n - n_0]}{2}.
\end{align*}
\]

Since \( y_1[n - n_0] \neq y_2[n] \), the system is Time Varying, or Not Time Invariant.

(b) The system completely removes the odd part of the signal and it cannot be reconstructed, so the system is not invertible. For a better explanation, let \( e[n] \) be an even signal, and consider

\[
\begin{align*}
x_1[n] &= e[n] + n \quad \rightarrow \quad y_1[n] = e[n] \\
x_2[n] &= e[n] - n \quad \rightarrow \quad y_2[n] = e[n]
\end{align*}
\]

Two distinct input signals go to the same output signal, i.e. \( x_1[n] \neq x_2[n] \) but \( y_1[n] = y_2[n] \), so the system is NOT Invertible.