1. Consider the signal

\[ x(t) = \begin{cases} 
0, & t < -\frac{1}{2} \\
2t + 1, & -\frac{1}{2} \leq t \leq \frac{1}{2} \\
2, & t > \frac{1}{2}
\end{cases} \]

(a) Use the differentiation and integration properties of the Fourier transform along with the Fourier Transform pair for a rectangular pulse to determine a closed-form expression for \( X(j\omega) \)

(b) What is the Fourier transform of

\[ g(t) = x(t) - 1 \]

2. (OW 4.25) Let \( X(j\omega) \) denote the Fourier transform of the following signal \( x(t) \)

Perform the following calculations without explicitly evaluating \( X(j\omega) \).

(a) Find \( \angle X(j\omega) \)

(b) Find \( X(j0) \)

(c) Find \( \int_{-\infty}^{\infty} X(j\omega) \, d\omega \)

(d) Evaluate \( \int_{-\infty}^{\infty} |X(j\omega)|^2 \, d\omega \)

(e) Sketch the inverse Fourier Transform of \( \Re \{X(j\omega)\} \)

3. (OW 4.36) Consider an LTI system whose response to the input

\[ x(t) = [e^{-t} + e^{-3t}] u(t) \]

is

\[ y(t) = [2e^{-t} - 2e^{-4t}] u(t) \]

(a) Find the frequency response of the system, \( H(j\omega) \).

(b) Determine the system’s impulse response, \( h(t) \).

(c) Find the differential equation relating the input and output of this system.
4. A causal and stable LTI system has frequency response

\[ H(j\omega) = \frac{4 + j\omega}{7 - \omega^2 + 8j\omega} \]

(a) Determine a differential equation relating the input, \( x(t) \), and output, \( y(t) \), of this system.

(b) Determine the impulse response \( h(t) \).

(c) What is the output when the input is

\[ x_1(t) = e^{-4t}u(t) \]

(d) What is the output when the input is

\[ x_2(t) = [e^{-4t} + te^{-4t}]u(t) \]

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**Additional Problems. Do Not Turn In!!**

5. (OW 4.31) This problem will illustrate the fact that the response to \( \cos(t) \) cannot be used to specify an LTI system uniquely.

(a) Show that the three LTI systems with impulse responses below all have the same response to \( x(t) = \cos(t) \).

\[
\begin{align*}
    h_1(t) & = u(t), \\
    h_2(t) & = -2\delta(t) + 5e^{-2t}u(t), \\
    h_3(t) & = 2te^{-t}u(t)
\end{align*}
\]

(b) Find the impulse response of another LTI system with the same response to \( \cos(t) \).

6. (OW 4.32) Consider an LTI system with impulse response

\[ h(t) = \frac{\sin[4(t-1)]}{\pi(t-1)} \]

Determine the system output for each of the following inputs:

(a) \( x_1(t) = \cos\left(6t + \frac{\pi}{2}\right) \)

(b) \( x_2(t) = \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k \sin(3kt) \)

(c) \( x_3(t) = \frac{\sin[4(t+1)]}{\pi(t+1)} \)