1. Consider a periodic signal \( y(t) \) with period \( T = 1 \) which is defined over one period as
\[
y(t) = 1 - 4|t| \quad \text{for } |t| \leq \frac{1}{2}
\]
(a) Derive the coefficients \( a_n \) for the Fourier Series expansion.
(b) Let \( y_N(t) \) be a truncated version of the Fourier Series defined as
\[
y_N(t) = \sum_{n=-N}^{N} a_n e^{j\omega_0 nt}.
\]
Use Matlab to plot \( y_N(t) \) for \( N = 1, 5, \) and 15.

2. Determine the Fourier series coefficients for the periodic signal shown below

3. Consider a periodic signal \( x(t) \) with period \( T = 2 \) which is defined over one period as
\[
x(t) = \exp(-t) \quad \text{for } -1 < t < 1.
\]
(a) Derive the coefficients \( a_n \) for the Fourier series expansion.
(b) Use Matlab to plot the truncated Fourier series \( x_N(t) \) for \( N = 1, 5, 15, \) and 25.

4. Consider signals \( x_1(t) \) and \( x_2(t) \) which are periodic with periods \( T_1 \) and \( T_2 \), respectively. Furthermore, \( x_1(t) \) has Fourier series coefficients \( a_n \) and \( x_2(t) \) has Fourier series coefficients \( b_n \). Define \( y(t) = \alpha x_1(t) + \beta x_2(t) \)
(a) Derive the Fourier series coefficients of \( y(t) \) if \( T_1 = T_2 = T \).
(b) What are the Fourier series coefficients of \( y(t) \) if \( T_1 = 2T_2 \).

5. In this problem, we will derive two very important properties of the continuous-time Fourier series: the multiplication property and Parseval’s relation. Consider periodic signals \( x(t) \) and \( y(t) \). Both signals have fundamental period \( T \), and Fourier series representations:
\[
x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 kt} \quad y(t) = \sum_{k=-\infty}^{\infty} b_k e^{j\omega_0 kt}
\]
(a) Show that the Fourier series coefficients of the signal
\[
z(t) = x(t)y(t) = \sum_{k=-\infty}^{\infty} c_k e^{j\omega_0 kt}
\]
are given by the discrete-time convolution
\[
c_k = a_k \ast b_k = \sum_{n=-\infty}^{\infty} a_n b_{k-n}
\]
(b) If \( y(t) = x^*(t) \), express \( b_k \) in terms of \( a_k \).

(c) Use the results of parts (a) and (b) to determine the Fourier series coefficients of \( |x(t)|^2 \) in terms of the Fourier series coefficients of \( x(t) \).

(d) Use the results of part (c) to prove that
\[
\frac{1}{T} \int_T |x(t)|^2 \, dt = \sum_{k=-\infty}^{\infty} |a_k|^2
\]

6. Consider a real periodic signal \( x(t) \) with period \( T \) and Fourier series representation:
\[
x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 kt}
\]

(a) Determine the Fourier series coefficients for the even part, i.e. \( x_e(t) = \mathcal{E} \{ x(t) \} \).

(b) Determine the Fourier series coefficients for the odd part, i.e. \( x_o(t) = \mathcal{O} \{ x(t) \} \).

7. (OW 3.24) Let \( x(t) \) be a periodic signal with fundamental period \( T = 2 \) and Fourier series coefficients \( a_k \)
\[
x(t) = \begin{cases} 
  t, & 0 \leq t \leq 1 \\
  2 - t, & 1 \leq t \leq 2 
\end{cases}
\]

(a) Determine the value for \( a_0 \).

(b) Determine the Fourier series representation for \( y(t) = \frac{d}{dt} x(t) \).

(c) Use the result of part (b) along with the differentiation property of the CTFS to determine the Fourier series coefficients of \( x(t) \).

8. (OW 3.26) Let \( x(t) \) be a periodic signal whose Fourier series coefficients are given by
\[
a_k = \begin{cases} 
  2, & k = 0 \\
  \left( \frac{1}{2} \right)^{|k|}, & \text{otherwise} 
\end{cases}
\]

Use Fourier series properties to answer the following questions:

(a) Is \( x(t) \) real?

(b) Is \( x(t) \) even?

(c) Is \( dx(t)/dt \) even?