CTFT Motivational Example

Let \( x(t) \) be a square wave of the form
\[
x(t) = \begin{cases} 
1 & |t| < T_1 \\
0 & |t| > T_1 
\end{cases}
\]

and let \( \tilde{x}(t) \) be its periodic extension with period \( T \). The Fourier series coefficients of \( \tilde{x}(t) \) are
\[
T a_k = \frac{2 \sin (k \omega_0 T_1)}{k \omega_0}
\]

These coefficients are plotted for several values of \( T \) and a fixed \( T_1 \) below. We note that as \( T \to \infty \) we get \( \tilde{x}(t) = x(t) \) and the coefficients give us the full envelope (in some sense).