ENEE244 Spring 1999
Homework Set Number 1 Solutions

February 5, 1999

1-1 List the first 16 numbers in base 12. Use the letters A and B to represent the last two digits.

Not much work to this problem- simply write down the answer:

$$0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, 10, 11, 12, 13$$

1-2 What is the largest binary number that can be obtained with 16 bits? What is its decimal equivalent?

The largest number in any base with n bits is the maximum digit repeated n times. In base 2 the maximum digit is 1 so the largest binary number is:

$$(1111111111111111)_2$$

To determine its decimal value you could do $2^{15} + 2^{14} + ... + 2^1 + 2^0$ and add them all up or realize that by adding one we get 2^{16}. The decimal equivalent is therefore:

$$(10000000000000000 - 1)_2 = 2^{16} - 1 = 65535$$

1-3 Convert the following binary numbers to decimal

$$\begin{aligned}
101110 &= 2^5 + 2^3 + 2^2 + 2^1 \\
&= 32 + 8 + 4 + 2 = 46 \\
1110101.11 &= 2^6 + 2^5 + 2^4 + 2^2 + 2^0 + 2^{-1} + 2^{-2} \\
&= 64 + 32 + 16 + 4 + 1 + 0.5 + 0.25 = 117.75 \\
110110100 &= 2^8 + 2^7 + 2^5 + 2^4 + 2^2 \\
&= 256 + 128 + 32 + 16 + 4 = 436
\end{aligned}$$

1-4 Convert the following numbers with the indicated bases to decimal

$$\begin{aligned}
(12121)_3 &= 1*3^4+2*3^3+1*3^2+2*3^1+1*3^0 \\
&= 81+54+9+6+1=151 \\
(4310)_5 &= 4*5^3+3*5^2+1*5^1 \\
&= 500+75+5=580 \\
(50)_7 &= 5*7^1 \\
&= 35 \\
(198)_{12} &= 1*12^2+9*12^1+8*12^0 \\
&= 144+108+8=260
\end{aligned}$$

1-5 Convert the following decimal numbers to binary

1231

$\frac{1231}{2}$=615 R=1	$\frac{615}{2}$=307 R=1	$\frac{307}{2}$=153 R=1	$\frac{153}{2}$=76 R=1	$\frac{76}{2}$=38 R=0	$\frac{38}{2}$=19 R=0
$\frac{19}{2}$=9 R=1	$\frac{9}{2}$=4 R=1	$\frac{4}{2}$=2 R=0	$\frac{2}{2}$=1 R=0	$\frac{1}{2}$=0 R=1	

Reading the digits off backwards we get $(1231)_{10} = (10011001111)_2$

673.23

Break it into its integer part (673) and its fractional part (0.23). First convert the integer part:

$\frac{673}{2}$=336 R=1	$\frac{336}{2}$=168 R=0	$\frac{168}{2}$=84 R=0	$\frac{84}{2}$=42 R=0	$\frac{42}{2}$=21 R=0	$\frac{21}{2}$=10 R=1
$\frac{10}{2}$=5 R=0	$\frac{5}{2}$=2 R=1	$\frac{2}{2}$=1 R=0	$\frac{1}{2}$=0 R=1		

So $(673)_{10} = (1010100001)_2$. Now the fractional part:

$$\begin{aligned}
0.23*2 &= 0.46 \\
0.46*2 &= 0.92 \\
0.92*2 &= 1.84 \\
0.84*2 &= 1.68 \\
0.68*2 &= 1.36 \\
0.36*2 &= 0.72
\end{aligned}$$

Since I am now tired of multiplying by two I write the answer as $(0.23)_{10} = (0.001110...)_2$.
Thus the overall answer is: $(1010100001.001110...)_2$

10^4

$\frac{10000}{2}$=5000 R=0	$\frac{5000}{2}$=2500 R=0	$\frac{2500}{2}$=1250 R=0	$\frac{1250}{2}$=625 R=0	$\frac{625}{2}$=312 R=1	$\frac{312}{2}$=156 R=0
$\frac{156}{2}$=78 R=0	$\frac{78}{2}$=39 R=0	$\frac{39}{2}$=19 R=1	$\frac{19}{2}$=9 R=1	$\frac{9}{2}$=4 R=1	$\frac{4}{2}$=2 R=0
$\frac{2}{2}$=1 R=0	$\frac{1}{2}$=0 R=1				

So $(10000)_{10} = (10011100010000)_2$

1998

$\frac{1998}{2}= 999$ R=0	$\frac{999}{2}= 499$ R=1	$\frac{499}{2}= 249$ R=1	$\frac{249}{2}= 124$ R=1	$\frac{124}{2}= 62$ R=0	$\frac{62}{2}= 31$ R=0
$\frac{31}{2}= 15$ R=1	$\frac{15}{2}= 7$ R=1	$\frac{7}{2}= 3$ R=1	$\frac{3}{2}= 1$ R=1	$\frac{1}{2}= 0$ R=1	

So $(1998)_{10} = (11111001110)_2$

1-6 Convert the following decimal numbers to the indicated bases

(a) 7562.45 to octal

First the integer part

$\frac{7562}{8}= 945$ R=2	$\frac{945}{8}= 118$ R=1	$\frac{118}{8}= 14$ R=6	$\frac{14}{8}= 1$ R=6	$\frac{1}{8}= 0$ R=1

And now the fractional part:

$$\begin{aligned} 0.45 * 8 &= 3.60 \\ 0.60 * 8 &= 4.80 \\ 0.80 * 8 &= 6.40 \\ 0.40 * 8 &= 3.20 \\ 0.20 * 8 &= 1.60 \end{aligned}$$

And our answer is $(7562.45)_{10} = (16612.3\overline{4631})_8$

(b) 1938.257 to hexadecimal

First the integer part

$\frac{1938}{16} = 121$	$\frac{121}{16} = 7$	$\frac{7}{16} = 0$
R=2	R=9	R=7

And now the fractional part:

$$\begin{aligned} 0.257 * 16 &= 4.112 \\ 0.112 * 16 &= 1.792 \\ 0.792 * 16 &= 12.672 = C.672 \\ 0.672 * 16 &= 10.752 = A.752 \\ 0.752 * 16 &= 12.032 = C.032 \end{aligned}$$

And our answer is $(1938.257)_{10} = (792.41CAC...)_{16}$

175.175 to binary

First the integer part:

$\frac{175}{2} = 87$	$\frac{87}{2} = 43$	$\frac{43}{2} = 21$	$\frac{21}{2} = 10$	$\frac{10}{2} = 5$	$\frac{5}{2} = 2$
R=1	R=1	R=1	R=1	R=0	R=1
$\frac{2}{2} = 1$	$\frac{1}{2} = 0$				
R=0	R=1				

And now the fractional part:

$$\begin{aligned} 0.175 * 2 &= 0.350 \\ 0.350 * 2 &= 0.700 \\ 0.700 * 2 &= 1.400 \\ 0.400 * 2 &= 0.800 \\ 0.800 * 2 &= 1.600 \\ 0.600 * 2 &= 1.200 \end{aligned}$$

Giving us an answer of $(175.175)_{10} = (10101111.001011...)_2$

1-7 Convert the hexadecimal number F3A7C2 to binary and octal

To get to binary we simply write each digit with 4 bits in binary:

$$(F3A7C2)_{16} = 1111\ 0011\ 1010\ 0111\ 1100\ 0010_2$$

To get to octal we regroup in sets of three bits:

$$111\ 100\ 111\ 010\ 011\ 111\ 000\ 010_2 = (74723702)_8$$

1-8 Convert the following numbers from the given base to the other three bases indicated

Under the assumption that you are familiar with the method I'll just give the answers here:

Decimal	Binary	Octal	Hexadecimal
225	11100001	341	E1
215	11010111	327	D7
403	110010011	623	193
10949	10101011000101	25305	2AC5

1-9 Add and multiply the following numbers without converting to decimal

The key here is to simply do all the math inside the indicated base. So:

(a) $(367)_8$ **and** $(715)_8$

$$\begin{array}{r} 367 \\ +\underline{715} \\ 1304 \end{array} \qquad \begin{array}{r} 367 \\ \underline{715} \\ 2323 \\ 3670 \\ \underline{330100} \\ 336313 \end{array}$$

(b) $(15F)_{16}$ **and** $(A7)_{16}$

$$\begin{array}{r} 15F \\ +\underline{A7} \\ 206 \end{array} \qquad \begin{array}{r} 15F \\ \underline{A7} \\ 999 \\ \underline{DB60} \\ E4F9 \end{array}$$

(a) $(110110)_2$ **and** $(110101)_2$

```
                              110110
                              110101
     110110                   110110
    +110101                 11011000
    1101011               1101100000
                         11011000000
                        101100101110
```

1-10 Perform the following division in binary

```
      110011
101 | 11111111
     -101
      0101
        101
        000111
           101
            0101
             101
              000
```