Dephasing time and one-dimensional localization of two-dimensional electrons in GaAs/Al$_x$Ga$_{1-x}$As heterostructures

K. K. Choi* and D. C. Tsui

Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08544

K. Alavi

Siemens Research and Technology Laboratories, Princeton, New Jersey 08540

(Received 2 September 1986; revised manuscript received 29 July 1987)

Low-field magnetoconductance measurements are made on the two-dimensional (2D) electron gas in GaAs/Al$_x$Ga$_{1-x}$As heterostructures to study the size effects on weak localization and the phase-breaking rate ($1/\tau_p$) as a function of sample width ($W$) and potential probe spacing ($L$). When $L$ and $W$ are larger than $\pi l_p$ and $\pi L_T$, where $l_p$ is the phase-breaking length and $L_T$ is the thermal diffusion length, $1/\tau_p$ deduced from experiment using the 2D localization theory, has some agreement with the theoretical 2D phase-breaking rate due to electromagnetic fluctuations and disagrees with the energy relaxation rate. When $W < \pi l_p$ and $\pi L_T$, both the localization effect and the determined $1/\tau_p$ are one dimensional. When $L$ is small and comparable to $l_p$, a new dimensional crossover for localization is observed. In such small samples, we also observe the predicted conductance fluctuations of order $e^2/h$.

Recent advances in the theory of weak localization have made it possible to determine the time scales for electron interactions in disordered systems. Such a possibility opens up a completely new way to study an interacting many-body system. With an applied magnetic field ($B$), various electron scattering time scales, such as those involved in spin-orbit scattering, spin-flip scattering, electron-phonon scattering, and electron-electron scattering, can readily be deduced. However, in order to extract any useful information, it is important to know the physical nature of the time scales deduced from the theory.

In the beginning stages of the theory, energy relaxation time ($\tau_e$) was believed to be the relevant parameter for the weak localization experiments. The physical picture has been modified in the more recent microscopic theory. The localization of electrons in the metallic regime, arising from the coherent interference among the electron waves scattered from randomly distributed impurities, is governed by the phase breaking time ($\tau_p$) of the electrons. These two times, $\tau_p$ and $\tau_e$, differ in a subtle way, although both are caused by inelastic scattering events. The dephasing time $\tau_p$ (Ref. 5) is defined as $\Delta \epsilon \tau_p \sim 1$, where $\Delta \epsilon$ is the variation of energy in time $\tau_p$. It depends on the inelastic scattering processes with large, as well as small, energy transfers. On the other hand, $\tau_e$ is defined as the time at which $\Delta \epsilon - \epsilon_0$, the energy of the electron relative to $E_F$. It depends primarily on the large energy transfer. Theoretical calculations show that $\tau_p$ and $\tau_e$ are significantly different in magnitude and that, in one dimension (1D), they have different functional dependences on $T$. These differences are within experimental reach and can be checked by experiments on the two-dimensional electron gas (2DEG) in semiconductors, as well as thin metal films. In this communication, we report our experimental data on the localization time scale of the 2DEG in GaAs/Al$_x$Ga$_{1-x}$As samples with different sample geometries and compare them systematically with the weak localization theory in different dimensions. Our results show that $\tau_p$ not $\tau_e$ is most likely the time scale relevant to the quantum interference giving rise to the weakly localized states in disordered systems.

The 2DEG in GaAs/Al$_x$Ga$_{1-x}$As heterostructures is an ideal system for testing the theory of dephasing time. First, due to the simple band structure of GaAs, it has a well-known 2D density of states, $N_0 = m^*/\pi \hbar^2$, and complications arising from the intervalley scatterings are avoided. Second, it has negligible spin-orbit coupling and is not known for having magnetic impurities. Consequently, the localization effect is determined purely by inelastic scattering processes, eliminating the need for additional adjustable parameters in the theoretical fitting of the data. Third, GaAs is a nonsuperconducting material, and the complications due to superconducting fluctuations are avoided. As a result, the experiment can be extended to a wide temperature range. In fact, our experiment was carried out as sufficiently low $T$ that phonon scattering is suppressed and electron-electron scattering dominates the inelastic process. In this low-$T$ regime, there are detailed theoretical calculations on $\tau_p$ and $\tau_e$ (Refs. 5, 9, 10, and 11), to allow direct quantitative comparison between theory and experiment. Our low-field ($< 140 \text{ G}$) magnetoresistance measurements are simultaneously done on channels with different widths, $W$, made from the same wafer, to demonstrate explicitly the difference between the 2D and 1D localization effects. The extracted $\tau_p$ is compared with the theory in the appropriate dimension. We also report the first direct observation of the effect of the potential probe spacing ($L$) on localization. When $L$ is comparable to $l_p \equiv \sqrt{D \tau_e}$, the observed magnetoresistance is reduced, indicating a new dimensional crossover. In addition, we will discuss the quantum fluctuations observed in the conductance of small samples.
We first recall that the localization correction \( \delta \sigma \) to the Drude conductivity in 2D is given by \(^1\)

\[
\delta \sigma_{2D} = -\frac{e^2}{2\pi^2 h} \ln \frac{\tau}{e} ,
\]

(1)

where \( \tau \) is the impurity scattering time. When a perpendicular \( B \) is applied, the time-reversal symmetry is broken and the localization effect is reduced according to \(^7\)

\[
\delta \sigma_{2D}(B) = -\frac{e^2}{2\pi^2 h} \left[ \psi \left( \frac{1}{2} + \frac{h}{4DeB}\tau_e \right) - \psi \left( \frac{1}{2} + \frac{h}{4DeB}\tau \right) \right] ,
\]

(2)

where \( \psi \) is the digamma function. Hence, the magnetoresistance is given by

\[
\frac{\Delta R(B)}{R} = -\frac{RW}{L} \left[ \delta \sigma_{2D}(B) - \delta \sigma_{2D}(0) \right] ,
\]

(3)

where \( R \) is the resistance at 4.2 K. Such a magnetic effect is a consequence of Landau condensation of the electron pair states in the Cooper channel.

When \( W \) is less than \( \pi l_e \), \(^12\) only the \( q_w = 0 \) states are important. (Here, \( q_w \) is the total momentum of the electron pairs along \( W \)). In this case, \( \delta \sigma \) is given by the 1D localization theory,

\[
\delta \sigma_{1D} = -\frac{e^2}{\pi h} \frac{1}{W} l_e ,
\]

(4)

In a weak \( B \), when \( W \leq l_e \equiv (\hbar/2eB)^{1/2} \), Landau condensation does not occur and the magnetoresistance is given by \(^13\)

\[
\frac{\Delta R(B)}{R} = -\frac{e^2}{L \pi h} \left[ \frac{1}{l_e^2} + \frac{W^2}{12l_e^2} \right]^{-1/2} - l_e ,
\]

(5)

where \( l_e = \sqrt{D\tau_e} \).

In order to verify Eqs. (3) and (5), four-terminal magnetoresistance measurements are performed from \( T = 4.2 \) to \( 0.3 \) K on samples with electron density \( N_e = 1.6 \times 10^{15} \) m\(^{-2} \), and mobility \( \mu = 27000 \) cm\(^2\)/V s. The corresponding electron diffusion constant \( (D) \) and \( \tau \) are \( D = 0.016 \) m\(^2\)/s and \( \tau = 1.0 \times 10^{-12} \) s. The electric field used in the measurement is less than 0.7 V/m to avoid electron heating. Our data from a wide sample \((L = 2100 \mu m, W = 300 \mu m)\) and a narrow sample \((L = 62 \mu m, W = 0.21 \mu m)\), made by using photolithographic techniques, are shown in Fig. 1. It is evident that both the magnitude and the functional form of the magnetoresistance of the two samples are different. They can be separately fitted using the 2D theory [Eq. (3)] and the 1D theory [Eq. (5)], respectively, in the entire \( T \) range. The solid curves are the theoretical fittings using a single adjustable parameter, \( 1/\tau_e \). Figure 2 shows the \( T \) dependences of \( 1/\tau_e \).

At low \( T \), when electron-electron interaction dominates, \( 1/\tau_e \) in 2D is given by \(^5,9,10\)

\[
\frac{1}{\tau_e} = \frac{\pi}{2} \left( \frac{kT}{E_F} \right) \ln \frac{E_F}{kT} , \quad \text{when } T > \frac{\hbar}{k\tau} ,
\]

\[
\frac{kT}{2\pi N_0D\hbar^2} \ln(\pi DN_0\hbar) , \quad \text{when } T < \frac{\hbar}{k\tau} .
\]

(6)

In Eq. (6), the \( T^2 \) term (with a weak \( \ln T \) dependence) is due to momentum conserving processes, whereas the \( T \) term is due to momentum nonconserving processes. For our samples, \( \hbar/k\tau \sim 7.6 \), indicating that both terms are comparable in our temperature range. Indeed, the data of

FIG. 1. The magnitude of the negative magnetoresistance for a wide channel (upper curves) and a narrow channel (lower curves) at different temperatures. The solid curves are the theoretical fittings based on Eq. (3) and Eq. (5), respectively. The fitting parameter, \( 1/\tau_e \), is plotted in Fig. 2.

FIG. 2. The phase breaking rate, determined from a wide channel (and a narrow channel (-\( \sigma \))), as a function of temperature. The solid curves are the best fittings based on Eq. (6) and Eq. (7).
the 2D sample in Fig. 2 can be fitted to a combination of 
$T^2$ and $T$ dependences (the solid curve), from which one
obtains
\[ \frac{1}{\tau_e} = 2.03 \times 10^{10} T^2 + 1.28 \times 10^{10} T \ (s^{-1}) \]
which agrees with Eq. (6) to within 12%. In a second 2D
sample with a similar $N_s$, $1.3 \times 10^{15} \text{ m}^{-2}$, but smaller $D$,
$-0.0090 \text{ m}^2/\text{s}$, we obtain
\[ \frac{1}{\tau_e} = 1.71 \times 10^{10} T^2 + 2.30 \times 10^{10} T \ (s^{-1}) \ . \]
According to Eq. (6), the $T$ term is inversely proportional
to $D$ and we have observed this dependence on $D$. We
should note that although the theoretical predictions on
the $T$ dependence of $\tau_e$ is similar to that of $\tau_e$ in 2D, the
magnitude of $\tau_e$ is a factor of 10 larger \cite{10} and cannot
fit our data at all.

For narrow channels, when $W < \pi L_T$ ($L_T \equiv \sqrt{hD/kT}$),
the momentum nonconserving process undergo a dimen-
sional crossover and lead to a $T^2$ dependence.\cite{3} The $T^2$
term, on the other hand, is not expected to change. Ac-
ccording to theory, $1/\tau_e$ in 1D is given by
\[ \frac{1}{\tau_e} = \frac{\pi}{2} \frac{(kT)^2}{h E_F} \ln \frac{E_F}{kT} + \left( \frac{kT}{D^{1/2} W N_0 \hbar^2} \right)^{2/3} . \] (7)
We have fabricated a total of four narrow channel
devices with $W$ ranging from 0.50 to 0.21 $\mu$m (Ref. 14) and
have studied their 1D magnetoresistance in the $T$ range
from 4.2 to 0.3 K. We found that the observed $T$
dependence of $1/\tau_e$ can indeed be fitted to a combination of $T^2$
and $T^{2/3}$ dependences. In Fig. 2, the squares show the $T$
dependence of $1/\tau_e$ from the $W=0.21 \mu$m sample of
Fig. 1 and the solid curve is the fit yielding
$1/\tau_e = 4.23 \times 10^{10} T^2 + 7.25 \times 10^{10} T^{2/3}$.
Several remarks are appropriate. First, the coefficient of the $T^{2/3}$ term
agrees with the theoretical value calculated from the
known sample parameters. In our samples, the agreement
is within 30% and is consistent with the expected dependence
on $W$. Second, the coefficient of the $T^2$ term is 2 to
3 times larger than that from theory, similar to what was
previously observed in the Si inversion layer.\cite{12} This
discrepancy between theory and experiment is genuine and
it need further theoretical investigation. Finally, it
should be noted that the observed $T$ dependence of $1/\tau_e$ in
1D can also be fitted to a $T^2$ and $T^{1/2}$ combination, as ex-
pected for the energy relaxation rate $1/\tau_e$. However, as in
the 2D case, the magnitude disagrees strongly with that
expected for $1/\tau_e$. In the case of the $W=0.21 \mu$m sample,
the fitted coefficient of the $T^{1/2}$ term is a factor of 4
larger, while that of the $T^2$ term remain approximately
the same.

We also investigated the dependence of the localization
effect on $L$. For a narrow channel with ideal current
contacts separated by $L$, the localization correction is\cite{15}
\[ \delta \sigma = -\frac{e^2}{\pi \hbar} \frac{1}{W} l_s \left( \coth \frac{L}{l_s} - \frac{l_s}{L} \right) . \] (8)
This expression can be rewritten in a more physically ex-
plicit form as
\[ \delta \sigma = -\frac{e^2}{\pi \hbar} \frac{1}{W L_{\text{eff}}} , \] (9)
where
\[ \frac{1}{L_{\text{eff}}} = \frac{1}{l_s^2} + \left( \frac{\pi}{L} \right)^2 . \]
(10)
Equation (9) agrees with Eq. (8) to within 5% in the en-
tire range of $L/l_s$ and it was indirectly confirmed, up to a
minor numerical factor, by Bishop and Dolan\cite{16} using
equal spacing doping techniques. The relation of Eq. (10)
is different from that for the electron-electron interac-
tions.\cite{17}

It turns out that a direct test of Eq. (9) is difficult. As the
channel is made narrower and shorter, the conductance
fluctuations, inversely proportional to $\sqrt{LW}$, be-
come more significant\cite{18} and prevent an accurate deter-
mination of the localization effect. In our experiment, $W$
is kept relatively wide, when the potential probe spacing is
decreased, to minimize these quantum fluctuations. This
geometry is sufficient to determine the length effect on
localization, and it can be shown, by changing variables,
that for this geometry with free boundary conditions,
insensitive to the shape of the sample. The magnetoresis-
tance measurements were made on a sample with $W=2.5$
$\mu$m and $L=8.0 \mu$m. The data fit the 2D magnetoresistance
theory from 4.2 to 0.3 K except for the fact that there are \sim 7% fluctuations superposed on the monotonic
background. $L_{\text{eff}}$, obtained from fitting the data, is plotted
as triangles ($\Delta$) in Fig. 3 along with $l_s$, obtained from a
long channel sample ($L=2100 \mu$m). At high $T$, when
$l_s \ll L$, the two curves merge, indicating that the effect of
$L$ is negligible. As $T$ decreases, $L_{\text{eff}}$ becomes substantially
smaller than $l_s$. The value of $L$ obtained from the data,
using Eq. (11), shows no $T$ dependence and is equal to 7.5
$\mu$m, consistent with the measured length to within 7%.

![FIG. 3. $L_{\text{eff}}$ ($\Delta$) from a short channel and $l_s$ ($\circ$) from a long channel as a function of $T$. The $\times$'s are $(L/2\pi)$, obtained from data using Eq. (11).]
Finally, Fig. 4 shows the magnetoresistance of the same short channel sample in $B$ up to 0.8 T at $T = 0.8$ K. It is apparent that random fluctuations are dominant. According to the theory of Lee and Stone, the magnitude of the fluctuations in conductance ($\Delta G$) is a universal constant $e^2/h$ at $T = 0$. At finite $T$, $\Delta G$ is reduced by a factor of either $\pi L_T/\sqrt{LW}$ (for $\eta = 1$) or $\pi L/\sqrt{LW}$ (for $\gamma = 1$), whichever is smaller. (The constants $\eta$ and $\gamma$ are defined in Ref. 16.) In our case, since both $\pi L_T$ (1.2 $\mu$m) and $\pi L$ (2.6 $\mu$m) are known, we can calculate $\Delta R = R^2 \Delta G(T)$ directly. We also estimate the correlation field, $B_c$, using $B_c A = 2.4h/e$ and $A$ is the smaller of $(\pi L_T)^2$ or $(\pi L)^2$. The values are $\Delta R = 370 \Omega$ and $B_c = 70$ G, which agrees well with the experiment.

In conclusion, we have investigated the effects due to sample geometry on localization by either narrowing $W$ or shortening $L$. The phase breaking rate, $1/\tau_p$, obtained from the experiments, agrees more closely with the theory of electromagnetic fluctuations in both 2D and 1D than $1/\tau_p$, does with the theoretical energy relaxation rate. This result shows that the phase breaking rate, not the energy relaxation rate, is most likely the relevant time scale in the weak localization theory. Recently, Thornton et al. deduced $\tau_p$ from a 450 $\AA$ channel in the range of $T$ from 0.4 to 1 K. Neglecting the contribution from the $T^2$ term, they found $1/\tau_p$ a factor of 12 larger than the theoretical prediction. One possibility of this discrepancy is the $T^2$ term, neglected in their work, is greatly enhanced in the much narrower channel. On the other hand, such a discrepancy raises a more fundamental question. In a 450 $\AA$ channel with $N_x = 4 \times 10^{15}$ $m^{-2}$, only two 1D subbands are occupied. The concept of diffusion along the width of the channel may not be valid in this extreme quantum regime. Since the 1D localization theory relies explicitly on the assumption of diffusion along the width, it may not be applicable to this extremely narrow channel. This issue casts some doubts on their deduced $\tau_p$. Finally, we show that the conductance fluctuations observed in our samples are consistent with the theory and these quantum fluctuations are expected to dominate over other physical phenomena in small samples.

The work at Princeton University is supported by the National Science Foundation through Grant No. DMR-8212167 and the U.S. Army Research Office.

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*Present address: AT&T Bell Laboratories, Murray Hill, NJ 07974.

1For a review, see for example, P. A. Lee and T. V. Ramakrishnan, Rev. Mod. Phys. 57, 287 (1985).


12R. G. Wheeler, K. K. Choi, A. Goel, R. Wisnieff, and D. E. Prober, Phys. Rev. Lett. 49, 1674 (1982). In this paper, $L_x$ is defined as $\sqrt{2}D_{x}$ in 2D and $\sqrt{D_{x}}$ in 1D.


