Analysis of a field-effect transistor with a channel made of ultrafine metal particles

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We calculate the characteristics of a type of field-effect transistor whose channel consists of ultrafine metal particles between the source and drain electrodes. The particles are capacitively coupled to a gate electrode through an insulating film. Electrons move between the source and the drain via the particles by tunneling. If the electrostatic energy required to add or to subtract an electron from a particle is much larger than the thermal energy, transitorlike behavior can be observed. The characteristics of the transistor are periodic with respect to the gate voltage. In principle, the proposed transistor has a subpicosecond carrier transit time and a transconductance of more than several Siemens per millimeter of channel width.

I. INTRODUCTION

Tunneling is of interest for transistor applications because of its short characteristic time (below $10^{-12}$ s). A transistor that makes use of tunneling in metal-insulator-metal (MIM) diodes has been proposed. MIM tunnel diodes were also studied to make wide-band detectors for the millimeter through infrared frequency range. A kind of MIM diode was constructed by depositing discontinuous metal films. The fine metal particles in the discontinuous film form many small MIM diodes. Although the electronic response time of the discontinuous films is very short, their application as transistors has not been reported.

The electronic conductivity $\sigma$ of discontinuous films shows the semiconductorlike temperature dependence, $\sigma \propto \exp(-E_g/kT)$. This temperature dependence is explained by using the theory of "activated tunneling" given by Neugebauer and Webb. The activation energy is the electrostatic energy required to add or to subtract an electron to or from a metal particle. Because of the activation energy, the films behave as if they have energy gaps. An electron that is thermally excited with an energy larger than the activation energy can move in the array of the particles without further excitation. Adkins et al. showed that the Neugebauer and Webb model predicts a field effect on the conductivity of discontinuous films. This prediction suggests to us that discontinuous films might be used as a kind of semiconductor. The experimentally observed field effects, however, were much smaller than that predicted by the simple theory given by Adkins et al. The discrepancy was attributed to the random distribution of particle size and the electrostatic potential to which the individual particles are subject. The potential disorder was also taken into consideration by Zeller and Giaever and Cavicchi and Silsbee to study conduction in discontinuous films. Although there will be many technical difficulties, we expect that the field effects in discontinuous films can be used in a transistor if the size of the particles and the potentials are well controlled.

In this paper we will present an analysis of a kind of a field-effect transistor (FET). The transistor is composed of metals, tunnel barriers, and gate insulators. The device utilizes the activated tunneling of electrons via ultrafine metal particles of dimensions of a few tens of nanometers.

II. ELECTRONIC CONDUCTION IN DISCONTINUOUS METAL FILMS

We will review the semiconductorlike properties that should appear in the idealized discontinuous film shown in Fig. 1. The metal particles are assumed to form a two-dimensional rectangular array. The size of each particle is assumed to be of the order of 100 Å, which is much larger than the de Broglie wavelength of electrons at the Fermi level of the particle. The sizes and shapes of each particle are identical. The particles are isolated from each other by the insulating medium between them. The electrons in a particle can move to neighboring particles via tunneling. The transmission probability of an electron incident upon the tunnel barrier is assumed to be much smaller than unity. Moreover, we assume that electron scattering in the particles and at the metal-insulator interfaces have a scattering length smaller than the periods $L_x$ and $L_y$, allowing us to neglect the superlattice effects in the structure illustrated in Fig. 1, although its periodicity is similar to that of the surface superlattice that is discussed by Bates, Ferry, Iafrate, Ferry, and Reich, and Reich et al. The transmission probability of the tunnel barrier is assumed to be small so that the quantum-mechanical uncertainty of energy of each particle can be neglected. This assumption requires an upper limit for the tunnel conductance as discussed later.

First, we will derive the carrier density of the film. The term "carrier" means an excited state of a particle that can propagate to a neighboring particle without further activation. The carrier density, or the probability of an excitation,
is derived by using the traditional capacitor model\(^6\) (Fig. 2), neglecting the interaction between excitations. In Fig. 2, \(C_g\) represents the capacitance between the gate electrode and the particle, and \(C'\) and \(G\) represent the capacitance and tunneling conductance from the particle to the ground electrode.

If we assume that there is no contact potential difference between the particle and the electrodes and that there is no fixed charge in the dielectrics of the capacitors, a flat-band condition holds. When the total charge of the particle is \(-ne\), the electrostatic energy of the system is \(U_a = n^2e^2/(2C) = n^2E_c\), where \(C = C' + C_g\) and \(E_c = e^2/(2C)\) is the activation energy. The electrostatic energy has a discrete value because the total charge is quantized in units of the charge of an electron. The value of \(E_g\) is a measure of the splitting of the electrostatic energies. We assume that the cutoff angular frequency of the circuit \(\omega_c = G/C\) satisfies \(\hbar\omega_c \ll E_c\) (i.e., \(G^{-1} \gg 8.2\, \text{kHz}\)). Thus the quantum-mechanical uncertainty of the energy due to tunneling is neglected compared to the splitting of the electrostatic energies.

The time spent by a tunneling electron in a barrier is as short as \(10^{-16}\ \text{s}\) and can be neglected. Tunneling is assumed to be elastic. When the number of excess electrons changes from \(n-1\) to \(n\) because of tunneling, the increase in electrostatic energy is \(U_a - U_{a-1}\). The energy supplied from the voltage source is \(\mu = EcV_g/C\). Thus an electron of energy \(E\) with respect to the Fermi energy of the ground electrode has an energy level of \(E - (U_a - U_{a-1}) + \mu\) with respect to the Fermi level of the particle after tunneling. The tunneling rate for electrons going from the ground electrode to the particle is approximated by the Fermi’s golden rule calculation with the use of the tunneling Hamiltonian as

\[
R_n = (4\pi/\hbar)N_C(0)N(0)|T|^2 \int f(E) \times [1 - f(E - E_n)] dE = (G/e^2)\{ - E_n/[1 - \exp(-E_n/kT)] \},
\]

where \(E_n = (U_a - U_{a-1}) - \mu, N_C(0)\) and \(N(0)\) are electronic density of states per spin in the ground electrode and the particle, respectively, and \(T\) is the matrix element of the tunneling Hamiltonian. The spacing of the energy levels in the particle due to quantum-mechanical size effects is neglected because it is of the order of 0.1 meV for a particle with a radius of 50 Å.\(^7\) The rate of the tunneling from the particle to the ground electrode with \(n\) changing to \(n-1\) because of tunneling is

\[
R_n = (G/e^2)\{ - E_n/[1 - \exp(-E_n/kT)] \}.
\]

Since tunneling is a stochastic process, the number of the excess electrons in the particle changes randomly with time. In a steady state we set the probability that the particle will have \(n\) excess electron as \(P_n\). The value of \(P_n\) is obtained by solving the following differential equations:

\[
P_{n-1}R_n + P_{n+1}R_{n+1} = P_n (R_{n+1} + R_n).
\]

Normalizing the solution of Eq. (2) gives

\[
P_n = \exp[ - (m^2 E_c - n\mu)/kT ]/ \sum_{-\infty}^{\infty} \exp[ -(m^2 E_c - m\mu)/kT ].
\]

If \(\mu = 2nE_c + \epsilon\) and \(E_c - |\epsilon| \gg kT\) holds, Eq. (3) reduces to

\[
P_{n+1} = \exp[ ( - E_c + \epsilon)/kT ],
\]

\[
P_{n-1} = \exp[ ( - E_c - \epsilon)/kT ],
\]

\[
P_n = 1 - P_{n+1} - P_{n-1}.
\]

The value of \(P_{n+1}\) is the probability of an electronic excitation and \(P_{n-1}\) of a hole-like excitation. It is clear from Eqs. (3) that the carriers densities are a periodic function of \(\mu\) or the applied gate voltage. The above results are the same as given by Adkins et al. with the use of statistical mechanics, but the “chemical potential” is explicitly related to the gate voltage \(V_g\). Carrier density per unit area of the discontinuous film is then given as

\[
n = P_{n+1}/(L_xL_y), \quad p = p_{n-1}/(L_xL_y),
\]

where \(L_x \times L_y\) is the area per particle. We see from Eq. (4) that the densities of carriers are controlled by the gate bias or by the field made by the gate electrode. If differences in contact potentials and fixed charges in dielectric films produce fields at the surface of the particle, the carrier densities will also change. “Doping” of the film may be possible by introducing these fields in a well-controlled manner. On the other hand, the field fluctuates because of fluctuation of contact potential differences and the random distribution of charged centers. It should be noted that if the statistical fluctuation of the potential of the particles is larger than the activation energy \(E_c\), the field effect will disappear.\(^6\)

The expression for the mobility of carrier is obtained following Neugebauer and Webb. We assume that an average electric field \(E = \bar{V}/L_x\) is in the x direction as shown in Fig. 1, where \(\bar{V}\) is a voltage between particles 1 and 2 when the both particles are neutral. Calculation of the tunneling rate between the particles gives an effective carrier velocity \(\bar{v}\), and then the mobility of the electron \(\mu_e\) is obtained. The result is

\[
\mu_e = \bar{v}/E = L_xG_0/e,
\]

where \(G_0\) is the tunnel conductance between neighboring particles. A similar calculation showed us that Einstein’s relation holds between the mobility and diffusion constant of the carrier when the carrier density is small. The mobilities of a hole-like excitation and an electronic excitation are the same. If \(1/G_0 = 50\, \text{kΩ}\) and \(L_x = 200\, \text{Å}\), the mobility is about 500 cm\(^2\)/Vs.

### III. ANALYSIS OF ACTIVATED TUNNELING FET CHARACTERISTICS

In the previous section we saw that discontinuous films behave as a kind of semiconductor. The carrier density of the film is controlled by field effect and doping. Both FET and

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bipolar transistor can be made of discontinuous metal films. A discontinuous film may be used for channel of a FET. In the limit of reduced channel length, the channel has only one particle between the source and drain. It is of great interest to note that the same transistor was proposed and analyzed recently by Likharev.\textsuperscript{15}

Figures 3(a) and 3(b) show a schematic diagram and an equivalent circuit of a field-effect transistor that has a small metal particle in the channel. Direct tunneling between a source and drain is assumed to be negligible when compared to the tunneling via a particle. In Fig. 3(b) $C_i$ and $G_i$ represent the capacitance and the conductance from the particle to the electrode $i$, where the subscript $i$ equals $s$, $d$, and $g$, corresponding to the source, drain, and gate, respectively. The electrodes are connected to voltage sources, and the electric potential of electrode $i$ is $V_i$.

Now we will calculate the probability that the charge on the particle is $-ne$. When the particle has $n$ excess electrons, the electrostatic energy of the channel $U_n$ is $(ne)^2/2C = n^2E_v$ where $C = \Sigma_i C_i$. When an electron tunnels from electrode $i$ to the particle and the number changes $n \rightarrow n$ to $n+1$, the energy supplied from voltage sources is $E = \Sigma_i (C_i V_i/C)e - eV_i$. The tunneling rate from the electrode $i$ to the particle is obtained by the method described in Sec. II and is

$$R_i(n) = \left(\frac{G_i}{e^2}\right) \left(\frac{(E_i - E_s)}{(E_i - E_n)/kT}\right) \{1 - \exp[-(E_i - E_n)/kT]\}, \quad (7a)$$

where

$$E_i = -eV_i, \quad E_n = (2n-1)e - \Sigma_j (C_j V_j/C)e.$$ 

The rate for tunneling from the particle to the electrode $i$ for $n$ changing from $n$ to $n-1$ is

$$R_i'(n) = \left(\frac{G_i}{e^2}\right) \left(\frac{(E_n - E_i)}{(E_n - E_i)/kT}\right) \{1 - \exp[-(E_n - E_i)/kT]\}. \quad (7b)$$

In Eqs. (7) the distribution function for the electrons in the particle is assumed to be the thermal equilibrium Fermi–Dirac distribution. The effect of hot electrons is neglected. The number of excess electron in the particle changes stochastically because of tunneling between the particle and the electrodes. The stochastic process is a Markoff process in which the tunneling rates are determined by the index $n$. Figure 4 is a state diagram, or a Shannon diagram, which represents the stochastic process in the transistor. Let $P_n$ denote the probability that the particle will have $n$ electrons. The rate equation for $P_n$ is obtained from Fig. 4 as follows:

$$\frac{d}{dt}P_n(t) = -[R_s(n+1) + R_d(n+1)]$$

$$+ [R_s'(n) + R_d'(n)]P_n(t)$$

$$+ [R_s(n) + R_d(n)]P_{n-1}(t)$$

$$+ [R_s'(n+1) + R_d'(n+1)]P_{n+1}(t). \quad (8)$$

In the above equation $G_s$, the tunneling conductance from the particle to the gate, is neglected compared to $G_s$ and $G_d$. All the characteristics of the proposed transistor can be deduced from Eq. (8). The current in the transistor is obtained by using the value of $P_n$, $I_n$ the current from the particle to the source, and $I_d$, the current from the drain to the particle, are given by

$$I_s = e\sum_n P_n(t) [R_s(n+1) - R_s'(n)]$$

$$I_d = e\sum_n P_n(t) [R_d'(n) - R_d(n+1)]. \quad (9)$$

In a steady state, by letting $(d/dt)P_n = 0$ and summing Eq. (8) for $n = -\infty$ to $n = m$, we obtain

$$[R_s(m+1) + R_d(m+1)]P_m$$

$$= [R_s'(m+1) + R_d'(m+1)]P_{m+1}. \quad (10)$$

The values of $P_n$ can be calculated from Eqs. (8) or (10) in conjunction with the normalization condition. It should be noted that $I_s = I_d = I$ holds for a steady state.

When the gate voltage changes from $V_g$ to $V_g^*$ with $V_g$ and $V_g^*$ kept constant, the values of $E_n$, $P_n$, and $I$ change to $E_n^*$, $P_n^*$, and $I^*$, respectively. For $V_g^* = V_g + e/C_s$, $E_n^* = E_{n-1}$ holds. Therefore, $R_s'(n) = R_d(n-1)$ and $P_n^* = P_{n-1}^*$ holds, and we get $I = I^*$ from Eq. (9). The current of the transistor is thus a periodic function of $V_g$ with period of $e/C_s$.

A. Characteristics at $T=0$

At $T = 0$ K, analytical expressions for the transistor characteristics are easily obtained from Eqs. (9) and (10). We set $V_g = 0$ without losing generality. Equations (7a) and (7b) tell that for values of $n$ with $E_n > 0$ and $E_n - E_d > 0$, the current change stochastically because of tunneling between the particle and the electrodes. The stochastic process is a Markoff process in

FIG. 3. (a) Schematic diagram of the FET with a particle between the drain and source. (b) Equivalent circuit of the FET.

FIG. 4. State diagram of the stochastic process in the transistor. In the figure $n$ represents the number of excess electrons in a particle. Each arrow indicates a change in the number of electrons during an infinitesimal time $dt$. 


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is also not a steady state. A nonzero steady-state probability $P_n$ exists for the value of $n$ which does not satisfy the above conditions. In $V_d - V_e$ space the conditions for $E_n = 0$ and $E_m - E_d = 0$ represent lines. In the each parallelogram made by these lines, there is a set of $n$ with nonzero $P_n$ as shown in Fig. 5.

When there is only one state that has a nonzero probability, the current in the transistor is zero because the exchange of electrons between electrodes and the particle does not occur. The transistor is in the “off” state. The transistor is driven to a current-carrying state by either increasing $V_d$ or applying an appropriate gate voltage $V_e$. It is interesting to note that there is a close resemblance between the proposed transistor and a dc-SQUID, or a two-junction interferometer made of Josephson junctions. The dc-SQUID is in a zero-voltage state when the bias current $I_b$ and the control current $I_c$ are in a certain area of an $I_b - I_c$ plane, and it is driven to a voltage state when the operating point goes out of the area. The similarity between Josephson junctions and a tunnel junction that contains ultrafine metal particles will be discussed elsewhere.

The current-voltage characteristics of the transistor are calculated for $V_d > 0$ and $V_e > 0$ in the mode where $n = 0$ and 1 gives a nonzero $P_n$. $P_n$ is obtained by using Eqs. (10) and the normalization condition. $I_d$ is given by

\[
I_d = G_x \left[ \frac{(C_x V_e + C_y V_d) / (C - E_e / \epsilon)}{(C_x + C_y) V_d / (C - C_y V_e / (C + E_e / \epsilon))} \right] + G_y \left[ \frac{(C_y V_e + C_d) / (C - E_e / \epsilon)}{(C_y + C_d) V_e / (C - C_y V_e / (C + E_e / \epsilon))} \right].
\]

The $I_d - V_d$ curves given by the above equation have asymptotes as follows:

\[
I_d = g V_d + g_m (V_e - V_{th}),
\]

where

\[
g = \frac{G_x G_e X (1 - X)}{(G_x G_e (1 - X) + G_x X)},
\]

\[
g_m = \frac{1 - 2X - g(G_y - G_x)}{G_x G_y} \frac{g(C_x / C)}{G_x G_y}.
\]

The turn-on voltage that is the maximum drain voltage with zero drain current is

\[
V_{on} = (V_e - V_{th}) C_x / C_d \quad \text{(for } V_e < V_{th})
\]

\[
= (V_e - V_{th}) C_x / (C_x + C_x) \quad \text{(for } V_e > V_{th}).
\]

An approximate value of the transconductance $g_m$ for $G_x = G_y = G$ and $C_x = C_d$ obtained from Eq. (12) is $g_m \approx (C_x / C)^2 G$. The voltage amplification factor is approximately given by $C_x / C_d$.

B. Characteristics at a finite temperature

The characteristics of the transistor have been calculated numerically. Figures 6(a)–6(g) show results for $G_x = G_y = 0.1 G_x$. As shown in Figs. 6(e)–6(g), the $I_d - V_d$ characteristics are a periodic function of $V_d$.

At drain voltages that are high compared with $E_e / \epsilon$, the gate loses its controlling characteristics, and the transconductance decreases. This is because the tunneling rates given by Eqs. (7a) and (7b) become approximately the same as the usual tunneling rate between large electrodes when $V_d \gg E_e / \epsilon$. The channel has a linear tunnel conductance that connects the source and the drain.

The transconductance also decreases as temperature increases. The activation energy required to add an electron to the particle is supplied thermally at high temperature when $kT \gg E_e$. The electrical characteristics of the channel then become that of a simple resistor. The activation energy is of the order of a few to a few tens of meV for a particle size of below 100 nm. Thus cryogenic temperature will be needed to operate the proposed transistor.

C. Switching speed

Transient characteristics of the transistor are analyzed by using the gate equation (8). We assume that $T = 0$ for simplicity. To calculate the intrinsic carrier transfer time of the transistor, the gate voltage $V_e$ is assumed to change stepwise at $t = 0$. First, we will derive the switch-on delay. In an initial state ($t < 0$) the transistor is in a “off” mode, and $P_0(t) = 1$ and $P_1(t) = 0$. The gate voltage is selected such that $P_n |_{n \neq 0, 1} = 0$ at $t \rightarrow \infty$. Equation (8) reduces to

\[
\frac{d}{dt} P_1(t) = - P_1(t) / \tau_{on},
\]

\[
\tau_{on} = \frac{e^2}{[G_d (E_0 - E_d) - G_e E_0]}
\]

Thus $P_1(t)$ as well as the drain current changes toward their final value with time constant $\tau_{on}$. The value of $\tau_{on}$ is $e^2 / G_d V_d$ for $G_y = G_x = G$. When $1/G = 50 \text{k}\Omega$ and $V_d = 20 \text{mV}$, $\tau_{on}$ is about 0.4 ps.


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When the transistor is switched off, the following equations hold:

\[
\left( \frac{d}{dt} \right) P_i(t) = -P_i(t)/\tau_{off},
\]

(14b)

\[
\tau_{off} = \frac{e^2}{[G_d(E_o - E_d) + G_iE_o].}
\]

For \(G_d = G_s = G\) and \(C_d = C_s\), \(\tau_{off}\) becomes

\[
\tau_{off} = \frac{e}{(GV)},
\]

\[
V = \left( \frac{C_s}{C_d} \right) [2(V_{th} - V_d) + V_d].
\]

(14c)

When \(1/G = 50 \text{ k}\Omega\) and \(V = 20 \text{ mV}\) \(\tau_{off}\) is also of the order of 0.4 ps.

When the transistors are integrated into a circuit, the switching delay is limited by the charging time of the capacitive load. Thus a large value for \(g_m\) is desirable. If a transistor with many particles in the channel is constructed from many single particle transistors connected in parallel (shown in Fig. 7), the value of \(g_m\) per unit channel width is approximately given by \(NG\), where \(G\) is the tunnel conductance between the particle and electrodes, and \(N\) is the number of particles per unit channel width. If \(N = 10^7/\text{mm}\) and \(1/G = 50 \text{ k}\Omega\) is possible, a transconductance of about 2 S/mm can be obtained.

**IV. CONCLUSION**

In this paper the characteristics of a FET that has a channel made of ultrafine metal particles has been analyzed. The number of electrons in a particle is a stochastic variable due to tunneling, and the change in this number was analyzed by using the standard method to treat a stochastic process. The analysis predicted transistorlike behavior with the assumption that the electrostatic energy required to move an electron or a hole to a particle is greater than the thermal energy \(kT\). The transistor will have a carrier transit time on the order of 0.5 ps and transconductance on the order of a few Siemens per millimeter.

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