Tunneling in a finite superlattice

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We have computed the transport properties of a finite superlattice from the tunneling point of view. The computed I-V characteristic describes the experimental cases of a limited number of spatial periods or a relatively short electron mean free path.

Since the original proposal and theoretical analysis of a one-dimensional superlattice having a period shorter than the electron mean free path, there have been considerable efforts to realize such a structure. Intriguing transport properties such as negative differential conductivity, Bloch oscillation, etc., in this man-made superlattice structure were predicted on the basis of a drastic reduction of the Brillouin zone into a series of minizones, due to the introduction of a large superlattice period.

The band model obviously assumes an infinite periodic structure. In reality, however, not only a finite number of periods is prepared with alternating epitaxy, but also the electron mean free path is relatively short. In addition, interfaces between the superlattice and terminal electrodes are unavoidable. For instance, GaAs-Ga1-xAlxAs superlattices grown on GaAs substrates are usually sandwiched between GaAs regions where Ohmic contacts are attached. Thus, the potential profile of the system is schematically illustrated in Fig. 1, where the right- and left-hand contacts correspond to pure GaAs. Periodically introduced Ga0.5Al0.5As layers give rise to regions having a barrier height of approximately 0.5 eV. The superlattice region extends over the range a distance l. With an applied voltage V, we assume that the field is uniformly distributed over a low-conducting superlattice region, as is shown in the lower part of Fig. 1, where E_T is the Fermi energy in the GaAs contact (E_T = 0.005 eV for n = 10^{17} cm^{-3}).

In order to understand the transport properties together with the role of the interfaces between the conducting GaAs terminals and the low-conducting superlattice, we apply the formalism of multibarrier tunneling to the system. By using the effective mass approximation for the unperturbed structures, the three-dimensional Schrödinger’s equation for the one-dimensional periodic potential represented by V(x), where x is along the direction of the multibarriers, may be separated into transverse and longitudinal parts, i.e., the total energy E is written by the sum of the longitudinal and the transverse energies,

E = E_L(V) + k^2 B_{k_x}^2/2m^* ,

and the wave function is expressed by the product,

\psi = \psi_L \psi_T .

For an n-period superlattice the electron wave functions in the left- and right-hand contacts are respectively

\psi_L = \psi_L \exp(ik_x l + R \exp(-ik_x l) ,

\psi_R = \psi_R \exp(ik_x l) ,

where R and T are the reflection and transmission amplitudes. By matching the derivatives and values at each discontinuity, we arrive at

\begin{equation}
\begin{split}
(M_1 \cdots M_p \cdots M_n) \frac{1}{R} ,
\end{split}
\end{equation}

where

\begin{equation}
\begin{split}
M_p = \frac{1}{4} \left( \exp(i k_{p-1} \frac{d_{p-2}}{2}) - \exp(-i k_{p-1} \frac{d_{p-2}}{2}) \right) \exp(-\frac{i k_{p+1}}{k_{p+2}} \frac{d_{p-1}}{2}) \frac{1 + i(k_{p-1}/k_{p+1})}{1 - i(k_{p-1}/k_{p+1})} ,
\end{split}
\end{equation}

in which k_p = [2m^*(V_p - E_T)^1/2]/\hbar .

In the above equations m^*, V_p, and E_T are the effective mass, the potential for the pth section, and the longitudinal energy, respectively.

The reflection and transmission amplitudes are given by

R = -M_{21}/M_{22}

and

T = M_{11} - M_{12}M_{21}/M_{22} .

To find the net tunneling current, we need to define the energy E which measures the energy of the incident electron and E', that of the transmitted. The current is given by

J = \frac{e}{4\pi \hbar} \int_0^\infty dk_x \int_0^\infty dk_y \left[ f(E) - f(E') \right] T^* T \frac{3E}{3k_y} .

Because of a separation of variables, the transmission coefficient T^* T is only a function of the longitudinal energy. Together with the Fermi distribution function, the expression for the current may be immediately integrated over the transverse direction, giving

J = \frac{e^2 m^* T}{2\pi \hbar^3} \int_0^\infty T^* T \ln \left( \frac{1 + \exp((E - E_T)/kT)}{1 + \exp((E - E_T - eV)/kT)} \right) dE_T .

For T \rightarrow 0, the above expression becomes

\begin{figure}[h]
  \centering
  \includegraphics[width=\textwidth]{figure1.png}
  \caption{Top: A finite superlattice of length l with barrier height \(\phi\). Below: Solid line indicates the potential profile used for our calculations with the application of an applied voltage \(V\).}
  \label{fig:figure1}
\end{figure}
is about twice the transmission peak of 0.082 eV. This is because the bottom of the well shifts only half the amount of the applied voltage for this case. Physically, the current maxima occur at certain voltages such that the resonant energies approach the Fermi energy of the electrons at the left of the barrier. The decrease in current with increase in applied voltages results from the fact that not only the energy but also the transverse momentum must be conserved in tunneling.

Because of the finite mean free path of the electrons, electrons may tunnel through $N$ periods before they encounter collisions, after which the coherency is destroyed. Most electrons will return to the lowest allowed states governed by statistics and the density of states. Under the influence of an applied voltage, these electrons will repeat the tunneling process through the next $N$ periods. The effect of incoherent multiple tunneling is to broaden the peaks primarily because of the spread in the electron mean free path and to increase the peak voltages by a factor in the $I$-$V$ characteristic, which is determined by the number of incoherent tunnelings. With a proper treatment of carrier statistics, this approach should, in principle, lead to the same results previously obtained with the band model. Nevertheless, any significant increase in the number of periods will result in rather tedious computations. Therefore, the band model approach should be more suitable for those cases with a long electron mean free path.

In conclusion, we have found that the multibarrier tunneling model provides a better insight to understand the observation of the transport mechanism in the case of a limited number of periods by design or a short mean free path in a superlattice.

**Figure 2.** Natural log of the transmission coefficient vs the electron energy in eV for the cases of a double-, triple-, and a quintuple-barrier. The barrier and well widths are 20 and 50 Å, respectively. The barrier height is 0.5 eV.

\[
J = \frac{\hbar}{m^*} \left( E_f - E_L \right) T^* T dE_f, \quad V > E_f, \\
\frac{\hbar}{m^*} \left[ \frac{\hbar}{m^*} - V \right] T^* T dE_f, \\
\frac{\hbar}{m^*} \left( E_f - E_L \right) T^* T dE_f, \quad V < E_f.
\]

This result may be applied to any number of barriers. Usually, in practice, the extent of the mean free path is only several periods. Therefore, we have computed a few cases up to five barriers.

Figure 2 shows the transmission coefficient $T^* T$ for a double-barrier (20 Å-50 Å-20 Å), a triple-barrier, and a quintuple-barrier case as a function of the electron energy. Note that the resonant energies for the triple-barrier case is split into doublets, and those for the quintuple-barrier case are split into quadruplets. The linewidths are roughly determined by the tunneling probability of the barrier width. For $n$ barriers, there will be an $(n-1)$-fold splitting. These resonant energies will eventually approach the band model of our original superlattice treatment for large $n$.

Figure 3 shows the calculated current density at 0°C for the double- and triple-barrier cases, without the constant factor $\frac{e \hbar}{m^*} / 2 \pi^2 \hbar$ as a function of the applied voltage. Note that the $I$-$V$ characteristics indicate fine structures having differential negative conductivities. The detail of these fine structures depends on the Fermi energy. However, this dependence is negligibly small for the case of a low Fermi energy which applies to most semiconductors. The first peak for the double-barrier case is located approximately at 0.16 V, which

**Figure 3.** Natural log of the current density $[\frac{e \hbar}{2 \pi^2 \hbar}]^{-1}$ vs the applied voltage $V$. $I_1$ and $I_2$ refer to the cases of a double- and triple-barrier, respectively.

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