ENEE 660 Fall 2010 Homework 6 (due in class Tuesday, October 26)

1. Obtain a reachability condition analogous to those given in Theorem (page 4, Lecture (3b)) for the matrix system

$$\dot{X}(t) = A(t)\,X(t) + X(t)\,B(t) + C(t)\,U(t)\,D(t)$$

where X is a square matrix of size $n \times n$, A, B are square matrices and the remaining matrices are of compatible sizes.

2. Let $Q = Q^T$ be a positive definite matrix. When is the linear time-invariant system

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & Q \\ -Q & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ B \end{bmatrix} u(t)$$

Controllable ?

3. Given the time-invariant system

$$\dot{x} = Ax + bu \qquad x(0) = 0,$$

and the constraint $u(t + \frac{1}{2}) = u(t)$

show that it is possible to select a control u such that $x(1) = x_1$ if and only if there exists a vector η such that

$$\left(e^{\frac{A}{2}} + I\right)\left(b, Ab, \dots, A^{n-1}b\right)\eta = x_1$$

4. Show that the <u>inverse</u> of the Gramian $W(t, t_1)$, if it exists, satisfies the nonlinear differential equation

$$\frac{d}{dt}P(t) = -A^T(t)\,P(t) - P(t)\,A(t) + P(t)\,B(t)B^T(t)\,P(t)$$

on the interval $[t_0, t_1]$ with <u>terminal</u> condition

$$P(t_1) = W(t, t_1)\Big|_{t=t_1} = 0$$