ENEE 660 Fall 2010 <u>Homework</u> #4 due in class Thursday, 09/30/2010

1. Let A be an $n \times n$ constant matrix. Use the properties of e^{tA} to derive the relation:

$$\mathcal{L}(e^{tA}) = (sI - A)^{-1}$$

where $\mathcal{L}$ denotes the Laplace transform. Use the Laplace transform to compute e^{tA} in Problem 6.2 of the textbook by

(a) hand, <u>without</u> using tables etc.

(b) MATLAB (see textbook).

2. Suppose that u and y are scalar functions of time satisfying

$$x^{(n)}(t) + p_{n-1} x^{(n-1)}(t) + \cdots + p_1 x^{(1)}(t) + p_0 x(t) = u(t)$$

$$y(t) = q_{n-1} x^{(n-1)}(t) + q_{n-2} x^{(n-2)}(t) + \cdots + q_0 x(t)$$

where $x^{(k)}(t)$ denotes the k^{th} derivative of $x(t)$, p_i, q_j are constants and $x^{(i)}(0) = 0$, $i = 0, 1, 2, \ldots, (n-1)$. Show that there exists a continuous function $w(\cdot)$ such that

$$y(t) = \int_0^t w(t-\sigma)\, u(\sigma)\, d\sigma .$$

Provide an expression for $w(t)$.

3. The adjoint differential equation associated to the system $\dot{x}(t) = A(t)\,x(t)$ is given by

$$\dot{p}(t) = -A^T(t)\,p(t),$$

where the superscript 'T' denotes the transpose of a matrix.

Show that $p^T(t)\,x(t) = p^T(t_0)\,x(t_0)$.

4. Let V be the space of $n \times n$ real matrices, and let $\mathcal{A} : V \to V$ be a linear map given by

$$\mathcal{A}(X) = M^T X + X M$$

where $M \in V$.

Derive the adjoint of $\mathcal{A}$.

Hint: First verify that $\mathrm{tr}(X_1^T X_2)$ defines an inner product on V.