ENEE631 Fall 2001
Lecture-4

Image Quantization (cont'd)

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Note: Smaller contribution from blue primary to luminance

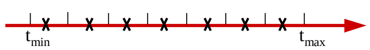
Review of Last Time

- Color coordinates
  - Linearly mixing 3 primary colors gives all color in the gamut
  - Different coordinates use different primaries and/or for different purposes
    - *differ by some linear transform* $\begin{bmatrix} Y \\ I \\ Q \end{bmatrix} = \begin{bmatrix} 0.2990 & 0.5870 & 0.1140 \\ 0.5960 & -0.2740 & -0.3220 \\ 0.2110 & -0.5230 & 0.3120 \end{bmatrix} \begin{bmatrix} R_N \\ G_N \\ B_N \end{bmatrix}$
- Image sampling and quantization
  - Get finitely many places to take samples & use finite bits per sample
  - Extend 1-D sampling to 2-D
    - *sampling (with comb function) vs. replication*
    - *aliasing and Nyquist rates*
    - *reconstruction by an ideal low pass filter*

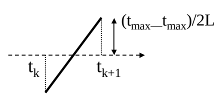
Review of Quantization

- L-level Quantization

  - Minimize errors for this lossy process
  - What L values to use?
  - Map what range of continous values to each of L values?

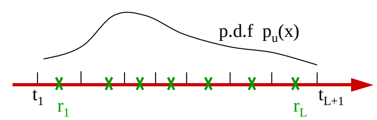


- Uniform partition
  - Maximum errors = (t_{max} - t_{min}) / 2L = A / 2L
    - dynamic range A
  - Best solution?
    - what if the value between [a, b] is more likely than other intervals?



quant. error

Bring in Probability Distribution



- Allocate more values in more probable ranges
- Minimize error in a probability sense
  - MMSE (minimum mean square error)

$$\begin{aligned} \mathcal{E} &= E[(u-u')^2] = \int_{t_1}^{t_{L+1}} (x-u'(x))^2 p_u(x)dx \\ &= \sum_{i=1}^{L} \int_{t_i}^{t_{i+1}} (x-r_i)^2 p_u(x)dx \end{aligned}$$

    - *assign high penalty to large error*
    - *squared error gives convenience in math.: differential, etc.*
- An optimization problem
  - what $\{t_k\}$ and $\{r_k\}$ to use?
  - Last time: necessary conditions by setting partial differential to zero

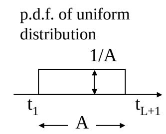
MMSE Quantizer (Lloyd-Max)

- Reconstruction and decision levels need to satisfy

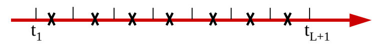
$$\begin{cases} t_k = \frac{r_k + r_{k-1}}{2} & \text{(midpoint between reconstruction levels)} \\ r_k = \frac{\int_{t_k}^{t_{k+1}} x p_u(x) dx}{\int_{t_k}^{t_{k+1}} p_u(x) dx} = E(u \mid u \in [t_k, t_{k+1})) & \text{(conditional expectation within decision interval)} \end{cases}$$

- Solve iteratively
  - Choose initial values of $\{t_k\}^{(0)}$, compute $\{r_k\}^{(0)}$
  - Compute new values $\{t_k\}^{(1)}$ and $\{r_k\}^{(1)}$ ……
- For large number of quantization levels
  - Approx. constant pdf within $[t_k, t_{k+1})$
  - Can obtain approximated closed-form solution

Quantizer for Uniform Distribution



- Uniform quantizer

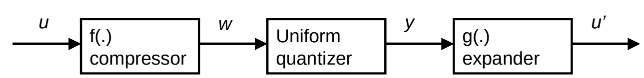


  - Optimal for uniform distributed r.v. in MMSE sense
  - MSE = q^2 / 12 with q = A / L
- SNR
  - Definition ……
  - Variance of uniform distributed r.v. = A^2 / 12
  - SNR = 20 $\log_{10}$ L = (20 $\log_{10}$2)*B = 6B (dB) with L = 2^B
    - *"1 bit is worth 6 dB."*

Think More About Uniform Quantizer

- Uniformly quantizing [0, 255] to 16 levels
  - where to partition and what 16 levels to use?
- Compare relative changes
  - x1 = 100 ∈ [96, 112) ➔ quantize to "104" ~ 4% change
  - x2 = 12 ∈ [0, 16) ➔ quantize to "8" ~ 33% change
  - Large relative changes could be easily noticeable by eyes!

Compandor



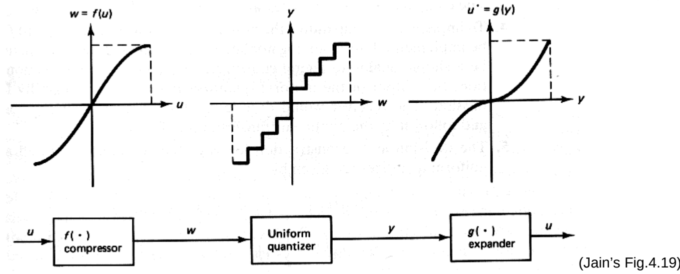
- Compressor-Expander
  - Uniform quantizer proceeded and succeeded by nonlinear transformations
    - *nonlinear func. amplifies or suppresses particular ranges*
- Why use compandor?
  - Inputs of smaller values suffer higher percentage of distortion under uniform quantizer
  - Nonlinearity in perceived luminance
    - *small difference in low luminance is more visible*

Compandor (cont'd)

- Nonlinear transformation functions
  - To make overall system approximate Lloyd-Max quantizer
  - Without iterative process to determine parameters

$$g(x) = f^{-1}(x)$$

$$f(x) = 2a \left\{ \frac{\int_{t_1}^{x} [p_u(\alpha)]^{1/3} d\alpha}{\int_{t_1}^{t_{L+1}} [p_u(\alpha)]^{1/3} d\alpha} \right\} - a$$



(Jain's Fig.4.19)

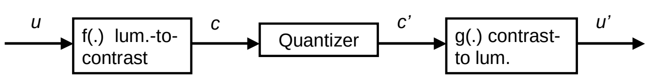
Visual Quantization

8 bits / pixel 4 bits / pixel 2 bits / pixel

- Contouring effect
  - Visible contours on smoothly changing regions for uniform quantized luminance values with less than 5-6 bits/pixel
    - *human eyes are sensitive to contours*
- How to reduce contour effect at lower bits/pixel?

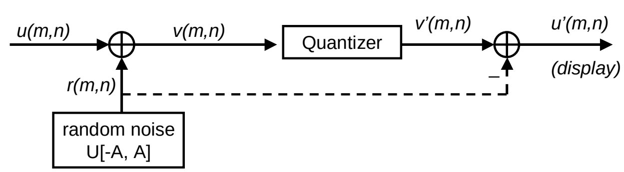
Tool-1: Contrast Quantization

- Visual sensitivity
  - non-uniform for luminance
  - almost uniform to just noticeable difference in contrast
    - *$\Delta L / L \sim 0.02$*
  - need 50 levels of contrast
    - *~ 6 bits with uniform quantizer*
    - *~ 4-5 bits with MMSE quantizer*
- Quantize contrast instead of luminance



Tool-2: Pseudorandom Noise Quantization

- Break contours (boundaries of const. gray area)
  - Add uniform zero-mean pseudo random noise before quantization *"dither"*
    - *keep average values unchanged*
  - Can achieve reasonable quality with 3-bit quantizer



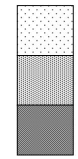
Example

4 bits / pixel

3 bits / pixel

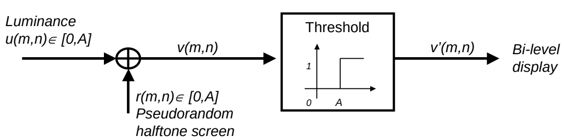
More on Pseudorandom Noise Quantization

- How about 1-bit quantizer?
  - Can we obtain a grayscale look with only black and white?
- Change density of black dots
  - spatial averaging function in human eyes gives a feel of grayscale by paying more attention on the average # of black dots per unit area



Halftone Images

- Binary images that give a grayscale look
- Extend the idea of pseudorandom noise quantizer
  - Upsample each pixel to form a resolution cell
  - Form dither signal by tiling halftone screen to be same size as the image
  - Apply quantizer on the sum of dither signal and image signal
- Perceived gray level
  - Equal to # of black dots perceived in one resolution cell



Examples of Halftone Images

- Applications
  - Photo printing in newspaper and books
  - Image display in the old black/white monitor
- Other approaches
  - Error diffusion

Color Quantization

- More sophisticated than simply quantizing RGB
  - Equal changes in color coordinates generally do not result in equal changes in perceived colors
- Carefully design transformation and quantizer to minimize the perceptual color difference

Image Transform

(Introduction)

Review: 1-D Signal & System *("Sig.&Sys" book, etc.)*

- Linear time-invariant system
  - Impulse response
- FS of periodic signal (continuous or discrete)
  - In terms of linear combination of harmonically related complex exponentials
- FT of 1-D continuous signal
- DTFT of sampled signal
  - Periodic replication in frequency domain
- DFT of 1-D discrete signal
  - fast algorithms

Review: 2-D Signal & System *(Jain's 2.2-2.5)*

- Linear Shift-invariance
  - 2-D impulse response and 2-D convolution
- Separable signal
  - f(x, y) = g(x) h(y)
- 2-D FT

$$\begin{cases} Z(\zeta_x,\zeta_y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} z(x,y)e^{-j2\pi(\zeta_x x+\zeta_y y)}\,dxdy \\ z(x,y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} Z(\zeta_x,\zeta_y)e^{j2\pi(\zeta_x x+\zeta_y y)}\,d\zeta_x d\zeta_y \end{cases}$$

  - Spatial frequency (normalized by viewing angle) ~ cycle per degree
  - Info. preserved (uniqueness): f(x,y) and its FT F(ξx, ξy)
  - Separable for each dimension => two successive 1-D FT along x & y
    - *what do we get if apply 2-D FT on a separable signal? (see Assign.-1)*

(time-domain) 1 Hz = 1 cycle per second
(spatial-domain) ~ cycles per unit distance
=> viewing angle matters! (small obj. viewing in small distance)

Summary

- Image Quantization
  - Optimal mean square quantizer
  - Uniform quantizer
  - Compandor
  - Visual quantization to reduce contour effects
    - *halftoning*
- Review of 1-D and 2-D signal and systems
- Next time
  - Image transform

Office Hour and TA

- Instructor's office hour
  - Thursday 3:30-5:30pm AVW 2457
  - Or by appointment ~ minwu@eng.umd.edu
- TA – Mr. Guan-Ming Su
  - Office hour:
    - *Tuesday 1-2pm (place TBA) and Friday 2-3pm Jasmine Lab*
    - *TA will be in Jasmine Lab on Friday 2-5pm in weeks with lab assignment*
  - Or by appointment ~ gmsu@glue.umd.edu

Assignment

- Readings
  - Jain's Sec.4.5-4.7, 4.12
- Assignment-1
  - Part-I (problem set) and Part-II (computer tasks)
  - 9/14/01 Friday 2-5pm Jesmine Lab (with TA)
    - *Attendance strongly recommended*
  - Submission → Due Wednesday 9/19/01 11:59pm
    - *Put your solution writeup (hard copy) in TA's mailbox*
    - *Put computer implementation and electronic results on your web page*
  - Reminder: decay function for late submission!