ENEE631 Fall 2001
Lecture-19

Image Filtering and Restoration

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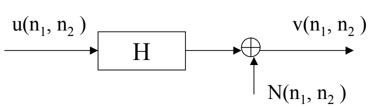
Last Time

- Geometrical manipulations
  - Described by matrix multiplication in homogeneous coordinates
  - Non-linear spatial warping
    - *Polynomial mapping and compensating lens distortions*
  - Implementation issues: forward vs. reverse mapping
- Image morphing
  - Feature-line based warping + dissolve
- Today
  - Image filtering and restoration

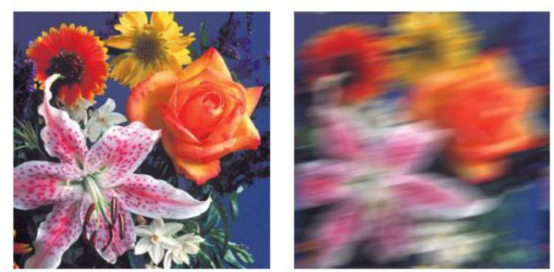
Imperfectness in Image Capturing



- Blurring ~ linear spatial-invariant filter model w/ additive noise
- Impulse response $h(n_1, n_2)$ & $H(\omega_1, \omega_2)$
  - Point Spread Function (PSF) ~ positive I/O
  - [No blur] $h(n_1, n_2) = \delta(n_1, n_2)$
  - [Linear translational motion blur]
    - *local average along motion direction*
  - [Uniform out-of-focus blur]
    - *local average in a circular neighborhood*
  - Atomspheric turbulence blur, etc.

$$h(x, y; L, \phi) = \begin{cases} \frac{1}{L} & \text{if } \sqrt{x^2 + y^2} \le L/2 \\ & \text{and } x/y = -\tan\phi \\ 0 & \text{otherwise} \end{cases}$$

$$h(x, y; R) = \begin{cases} \frac{1}{\pi R^2} & \text{if } \sqrt{x^2 + y^2} \le R^2 \\ 0 & \text{otherwise} \end{cases}$$



Original image   Blurred image

From Matlab ImageToolbox
Documentation pp12-4

Fourier Transf. of PSF for Common Distortions

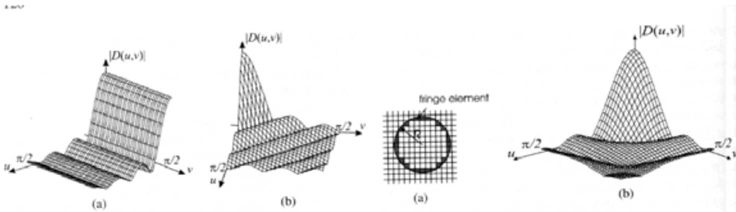


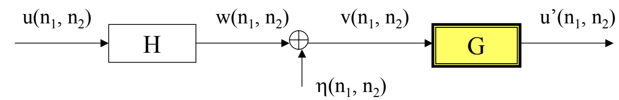
FIGURE 2 PSF of motion blur in the Fourier domain, showing $|D(u, v)|$, for (a) $L = 7.5$ and $\phi = 0$; (b) $L = 7.5$ and $\phi = \pi/4$

FIGURE 3 (a) Fringe elements of discrete out-of-focus blur that are calculated by integration; (b) PSF in the Fourier domain, showing $|D(u, v)|$, for $R = 2.5$.

From Bovik's Handbook
Sec.3.5 Fig.2&3

Undo Linear Spatial-Invariant Distortion

- Assume the PSF of distortion $h(n_1, n_2)$ is known



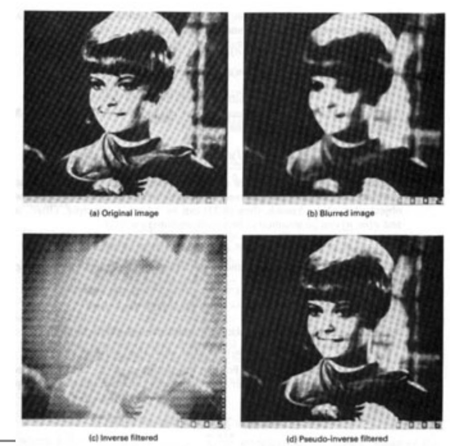
- Deconvolution / Inverse-Filtering
  - Often used for deblurring
  - Want to find $g(n_1, n_2)$ satisfies $h(n_1, n_2) \otimes g(n_1, n_2) = \delta(n_1, n_2)$
    - $\Sigma\Sigma\ h(k_1, k_2)\, g(n_1-k_1, n_2-k_2) = \delta(n_1, n_2)$ *for all* n_1, n_2
  - Easy to solve in spectrum domain $G(\omega_1,\omega_2) = \dfrac{1}{H(\omega_1,\omega_2)}$
    - *Convolution ⇔ Multiplication*
    - $H(\omega_1, \omega_2)\, G(\omega_1, \omega_2) = 1$
    - *Interpretation: choose G to compensate distortions from H*

Problems With Inverse Filtering

- Zeros in $H(\omega_1, \omega_2)$
  - Interpretation: distortion by H removes entire info. in those freq. band
  - Inverse filter tries to "compensate" by assigning infinite gains
  - Amplifies noise
    - $U'(\omega_1, \omega_2) = (W+N) / H \approx U + N/H$
- Solutions ~ Pseudo-inverse Filtering
  - Assign zero gain for G at spectrum nulls of H
  - Interpretation: not bother to make impossible compensations

$$G(\omega_1,\omega_2) = \begin{cases} \dfrac{1}{H(\omega_1,\omega_2)}, & \text{if } |\mathrm{H}(\omega_1,\omega_2)| \geq \varepsilon \\ 0, & \text{if } |\mathrm{H}(\omega_1,\omega_2)| < \varepsilon \end{cases}$$

Examples of Inverse & Pseudo-inverse Filtering



(a) Original image (b) Blurred image (c) Inverse filtered (d) Pseudo-inverse filtered

Figure 8.10 Inverse and pseudo-inverse filtered images.

From Jain Fig.8.10

Handling Noise in Deconvolution

- Inverse and pseudo-inverse filtering is sensitive to noise
- Try to balance between deblurring vs. noise suppression
  - Minimize MSE between the original and restored
    - $e = E\{ [u(n_1, n_2) - u'(n_1, n_2)]^2 \}$ *where* $u'(n_1, n_2)$ *is a func. of* $\{v(m_1, m_2)\}$
  - Best estimate is conditional mean $E[u(n_1, n_2) \mid \text{all } v(m_1, m_2)]$
    - *satisfy orthogonal condition* $E\{ [u(n_1, n_2) - u'(n_1, n_2)] v(m_1, m_2) \}=0$
    - *usually difficult to solve for general restoration (need conditional prob.)*
- Get the best linear estimate instead ➔ Wiener filtering

Wiener Filtering

- Get the best linear estimate minimizing MSE
  - Assume spatial-invariant filter $u'(n_1, n_2) = g(n_1, n_2) \otimes v(n_1, n_2)$
  - Assume wide-sense stationarity for original signal and noise
  - Assume noise is uncorrelated with original signal
- Solutions
  - Bring into orthogonal condition $E\{ [u(n_1, n_2) - u'(n_1, n_2)] v(m_1, m_2) \}=0$
  - Represent in correlation functions $R_{uv}(k,l) = g(k,l) \otimes R_{vv}(k,l)$
  - Take DFT to get representation in power spectrum density

$$G(\omega_1,\omega_2) = \frac{S_{uv}(\omega_1,\omega_2)}{S_{vv}(\omega_1,\omega_2)} = \frac{H^*(\omega_1,\omega_2) S_{uu}(\omega_1,\omega_2)}{|H(\omega_1,\omega_2)|^2 S_{uu}(\omega_1,\omega_2) + S_{\eta\eta}(\omega_1,\omega_2)}$$

$$\text{Note: } \begin{cases} S_{vv}(\omega_1,\omega_2) = |H(\omega_1,\omega_2)|^2 S_{uu}(\omega_1,\omega_2) + S_{\eta\eta}(\omega_1,\omega_2) \\ S_{uv}(\omega_1,\omega_2) = H^*(\omega_1,\omega_2) S_{uu}(\omega_1,\omega_2) \end{cases}$$

More on Wiener Filtering

- Balancing between two jobs for deblurring noisy image
  - HPF filter for de-blurring (undo H distortion)
  - LPF for suppressing noise

$$G(\omega_1,\omega_2)_{\text{wiener}} = \frac{1}{H + \dfrac{S_{\eta\eta}}{H^* S_{uu}}}$$

- Noiseless case ~ $S_{\eta\eta} = 0$
  - Wiener filter becomes pseudo-inverse filter

$$G(\omega_1,\omega_2)_{\text{wiener}} \Big|_{S_{\eta\eta}\to 0} = \frac{1}{H + S_{\eta\eta}/S_{uv}} \Big|_{S_{\eta\eta}\to 0} = \begin{cases} \dfrac{1}{H(\omega_1,\omega_2)}, & \text{if } |H(\omega_1,\omega_2)| \neq 0 \\ 0, & \text{if } |H(\omega_1,\omega_2)| = 0 \end{cases}$$

- No-blur case ~ $H = 1$ (Wiener Smoothing Filter)
  - Attenuate noise according to SNR at each freq. band

$$G(\omega_1,\omega_2)_{\text{wiener}} \Big|_{H=1} = \frac{S_{uu}(\omega_1,\omega_2)}{S_{uu}(\omega_1,\omega_2) + S_{\eta\eta}(\omega_1,\omega_2)} = \frac{S_{SNR}(\omega_1,\omega_2)}{S_{SNR}(\omega_1,\omega_2) + 1}$$

Comparisons

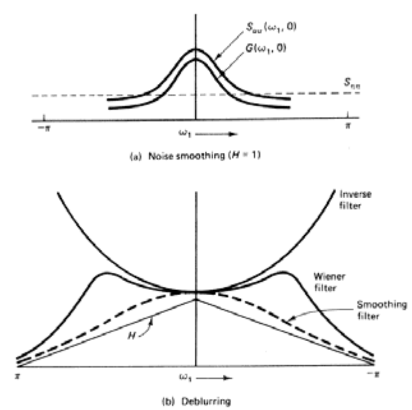


Figure 8.11 Wiener filter characteristics.

From Jain Fig.8.11

Issues to Be Addressed

- Wiener filter's size
  - Infinite impulse response ~ require large-size DFTs
  - Impose filter size constraint and find the best FIR minimizing MSE
- Need to estimate power spectrum density of orig. signal
  - Estimate p.s.d. of blurred image *v* and compensate variance due to noise
  - Estimate p.s.d. from a set of representative images similar to the images to be restored
  - Assume statistical model for the orig. image and estimate parameters
    - *e.g., 2-D AR model*
  - Constrained least square filter ~ see Jain's Sec.8.8 & Bovik's pp132
    - *avoid estimating p.s.d.*
    - *restrict (1) differences between blurred image and blurred version of reconstructed image under, and (2) smoothness in restored image*
- Unknown distortion *H* ~ Blind Deconvolution

Basic Ideas of Blind Deconvolution

- Estimate *H* via spectrum's zero patterns
  - Two major classes of blur (motion blur and out-of-focus)
  - *H* has nulls related to the type and the parameters of the blur
- Maximum-Likelihood blur estimation
  - Each set of image model and blur parameters gives a "typical" blurred output
  - Probability comes into picture because of the existence of noise
  - Given the observation of blurred image, try to find the set of parameters that is most likely to produce that blurred output
  - Iteration ~ Expectation-Maximization approach (EM)
    - *Given estimated parameters, restore image via Wiener filtering*
    - *Use restored image to refine parameter estimation*
    - *Get local optimums*
- To explore more
    - *Bovik's Handbook Sec.3.5 (subsection-4, pp136)*
    - *"Blind Image Deconvolution" by Kundur-Hatzinakos, IEEE Sig. Proc. Magazine, vol.13, 1996*

Filtering Through Transform Domain Operation

- Realize Wiener in DFT domain
- Use zonal mask in transform domain
  - Realize "ideal" LPF/BPF/HPF
  - Computation complexity for transf. could be high for large image

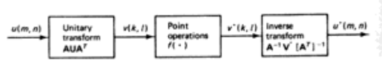


Figure 7.31 Image enhancement by transform filtering.

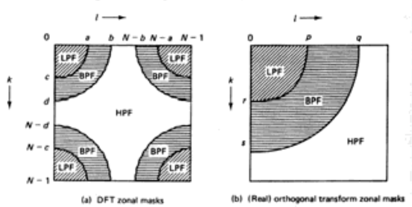


Figure 7.32 Examples of zonal masks $g(k, l)$ for low-pass filtering (LPF), band-pass filtering (BPF), and high-pass (HPF) filtering in (complex) DFT and (real) orthogonal transform domains. The function $g(k, l)$ is zero outside the region of support shown for the particular filter.

From Jain Fig.7.31&7.32

Summary

- Inverse filtering and pseudo-inverse filtering
- Wiener filtering to minimize restoration MSE
- Transform-domain operations
- Next time
  - Enhancement and restoration issues for video

Assignment

- Readings
  - Jain's book Sec.2.11-12, Sec.7.5, Sec.8.1-8.4
  - Bovik's Handbook Sec.3.5
- Reference – tutorials/surveys in IEEE Sig.Proc. Magazine
  - "Digital Image Restoration" by Banha-Katsaggelos, vol.14, 1997
  - "Blind Image Deconvolution" by Kundur-Hatzinakos, vol.13, 1996
- Project proposal
  - Due 11/13/2001 Tuesday 1:59pm
  - Submit electronically or a hard copy
- TA office hour
  - Friday 11/9 2-3pm

Clarification and Typo Corrections for Lec.18

- General 8-parameter matrix operations under homogeneous coordinate system

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \sim \begin{bmatrix} sx' \\ sy' \\ s \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Dissolve … typo
  - Should be $I_t = (1\text{-}t)\, I_0 + t\, I_1$

Compensating Spatial Distortion in Imaging

- Control points
  - Coordinates before and after distortion are known
- Fit into polynomial warping model
  - Minimize the sum of squared error between a set of warped control points and the polynomial estimates
    - $x' = [\, x_1, x_2, \ldots, x_M \,]$, $Z = [\, 1,\ x_1,\ y_1,\ \ x_1^2, x_1 y_1,\ y_1^2\ ;\ 1,\ x_2, y_2, \ldots \,]$
    - $E = (\, x' - Z\, a\,)^T (\, x' - Z\, a\,) + (\, y' - Z\, b\,)^T (\, y' - Z\, b\,)$
    - $\partial E \,/\, \partial a = 0 \ \Rightarrow x' = Z\, a$
  - Least square estimates
    - *Solution expressed by generalized inverse*
    - $a = Z^{\wedge} x' = (Z^T Z)^{-1} Z^T x'$
    - $b = Z^{\wedge} y'$
- Higher-order approximation
  - 2nd order polynomial usually suffices for many applications