University of Maryland at College Park
Department of Electrical and Computer Engineering

ENEE 624 Advanced Digital Signal Processing

# **Problem Set 7**

Spring 2003

**Issued:** Wednesday, April 23, 2003 **Due:** Monday, May 5, 2003

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**Reading Assignment:** Lectures 21–23 (*Hayes*, Chapter 7: Sections 7.1-7.2.3, and 7.3.

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**Exercise 7.A** (do not turn in)

*Hayes*, Problems 7.1, 7.9, and 7.11.

**Problem 7.1**

*Hayes*, Problem 7.4.

**Problem 7.2**

*Hayes*, Problem 7.7.

**Problem 7.3**

*Hayes*, Problem 7.14.

**Problem 7.4**

*Hayes*, Problem 7.10, parts (a), (b), and (c).

**Problem 7.5**

*Hayes*, Problem 7.8, parts (a) and (b).

**Problem 7.6**

Let $\mathsf{x}$ be an $N$-dimensional zero-mean random vector whose covariance matrix has eigenvalues

$$\lambda_1 > \lambda_2 > \cdots > \lambda_N\,,$$

and corresponding eigenvectors

$$\phi_1,\, \phi_2,\, \cdots,\, \phi_N\,.$$

Suppose we wish to approximate $\mathsf{x}$ as a (scalar) random variable $\mathsf{b}$ times a deterministic $N$-dimensional vector $\mathbf{a}$ (*i.e.*, $\mathsf{b}\,\mathbf{a}$). We are interested in finding the "best" $\mathsf{b}$ and $\mathbf{a}$, so that $\mathsf{b}\,\mathbf{a}$ is as "close" (in some appropriate sense) to $\mathsf{x}$ as possible.

(a) Given a realization $\mathsf{x} = \mathbf{x}$, determine

$$b_{\text{opt}} = \arg\min_{b\in\mathbb{R}} ||\mathbf{x} - b\,\mathbf{a}||\,,$$

where $||\cdot||$ is the Euclidean norm on $\mathbb{R}^N$, *i.e.*, $||\mathbf{z}|| = \mathbf{z}^T\mathbf{z}$. Find an explicit expression for $b_{\text{opt}}(\mathbf{x})$ in terms of $\mathsf{x}$ and $\mathbf{a}$. Note that, with $\mathsf{x}$ viewed as a random variable, the optimal $\mathsf{b}_{\text{opt}}$ is also a random variable.

(b) Let $\mathsf{b}_{\text{opt}}$ be as defined in part (a). Now define

$$\mathbf{a}_{\text{opt}} = \arg\min_{\mathbf{a}\in\mathbb{R}^N} E\left[||\mathsf{x} - \mathsf{b}_{\text{opt}}\,\mathbf{a}||^2\right]\,.$$

Show that Show that

$$\mathbf{a}_{\text{opt}} = \arg\max_{\mathbf{a}\in\mathbb{R}^N} \frac{\operatorname{var}\mathbf{a}^T\,\mathsf{x}}{\mathbf{a}^T\,\mathbf{a}}\,.$$

(c) Determine

$$\max_{\mathbf{a}\in\mathbb{R}^N} \frac{\operatorname{var}\mathbf{a}^T\,\mathsf{x}}{\mathbf{a}^T\,\mathbf{a}}$$

and indicate the value(s) of $\mathbf{a}$ for which the maximum is achieved.

(d) Repeat part (c) when we impose the constraint that

$$\mathbf{a} \perp \phi_i \quad (i.e., \mathbf{a}^T\phi_i = 0 \text{ for } i = 1,\, 2,\, \cdots .\, k-1),$$

for some $k \geq 2$. Again, indicate the value(s) of $\mathbf{a}$ for which the maximum is achieved.