University of Maryland at College Park
Department of Electrical and Computer Engineering

ENEE 624 Advanced Digital Signal Processing

Problem Set 5

Spring 2003

Issued: Wednesday, April 2, 2003 **Due:** Monday, April 14, 2003

Reading Assignment: Lectures 12–16 (*Hayes* , Chapter 3, Sections 3.0-3.7; Chapter 4, Sections 4.1–4.4 and 4.6–4.8).

Exercise 5.A (do not turn in)

Hayes, Problems 3.3, 3.4, 3.8, and 3.14.

Exercise 5.B (do not turn in)

Hayes, Problem 4.2.

Problem 5.1

Consider a real-valued process $x[n]$ for which $x[0] = 0$, and which satisfies the first-order AR model equation

$$x[n] + a[1]\, x[n-1] = v[n],$$

where $a[1]$ is a real-valued constant and $v[n]$ is a real-valued stationary white noise process with mean m_v and variance σ_v^2. Clearly, since $x[0] = 0$, the process $x[n]$ is not wide-sense stationary (except in the trivial case that $x[n] = 0$ for all n).

(a) Determine the mean $m_x[n]$ of the process $x[n]$ for $n > 0$.

(b) Determine whether or not $x[n]$ is asymptotically stationary in the mean. (A process $x[n]$ is asymptotically stationary in the mean if the mean $m_x[n]$ becomes independent of n as $n \to \infty$, *i.e.*, if $\lim_{n\to\infty} m_x[n]$ exists and is finite.)

(c) Determine the variance $\mathrm{var}(x[n])$ of the process $x[n]$ for $n > 0$ in the case that $v[n]$ is zero-mean.

(d) Determine whether or not $x[n]$ is asymptotically wide-sense stationary in the case that $v[n]$ is zero-mean. (A process $x[n]$ is asymptotically wide-sense stationary if its mean $m_x[n]$ becomes independent of n and its autocorrelation function $r_x[n; n-k]$ depends only on the lag k as $n \to \infty$.)

Problem 5.2

Hayes, Problem 4.5.

Problem 5.3

Hayes, Problem 4.7.

Problem 5.4

Hayes, Problem 4.13.

Problem 5.5

Hayes, Problem 4.19.

Problem 5.6

Hayes, Problem 4.21.

Problem 5.7

Consider the problem of modeling the finite-duration signal

$$x[n] = \delta[n] - \alpha\delta[n-1]$$

via the impulse response of an all pole-model $A_p(z)$, where the filter parameters in $A_p(z)$ are to be selected by means of the autocorrelation method. We are interested in the two cases $|\alpha| < 1$ and $|\alpha| > 1$.

(a) Determine the set of equations that must be satisfied by the parameters $a_p[k]$ of the pth-order all-pole model $A_p(z)$ selected via the autocorrelation method. In addition, verify that for $|\alpha| \neq 1$,

$$a_p[k] = \alpha^k \frac{1 - \alpha^{2(p-k+1)}}{1 - \alpha^{2(p+1)}},$$

for $k = 1, 2, \cdots, p$ is the solution to these equations. Finally, determine the associated squared error ϵ_p.

(b) Determine the limit of the all-pole model $A_p(z)$ and the associated squared error ϵ_p as $p \to \infty$, in the two cases $|\alpha| < 1$ and $|\alpha| > 1$.