University of Maryland at College Park
Department of Electrical and Computer Engineering

ENEE 624 Advanced Digital Signal Processing

Problem Set 4
Spring 2003

Issued: Wednesday, March 5, 2003 **Due:** Wednesday, March 12, 2003

Reading Assignment: Lectures 9–11 (optional: *Vaidyanathan*, Chapter 5, Sections 5.0-5.7.1, 5.8).

Problem 4.1

Consider the two-channel QMF bank in Fig. 4.1–1 with analysis and synthesis filters given by

$$H_0(z) = 3 + 4\,z^{-1} - 3.5\,z^{-2} + z^{-3} + z^{-4}, \qquad H_1(z) = H_0(-z)\ .$$

Find a set of causal and stable synthesis filters that result in perfect reconstruction.

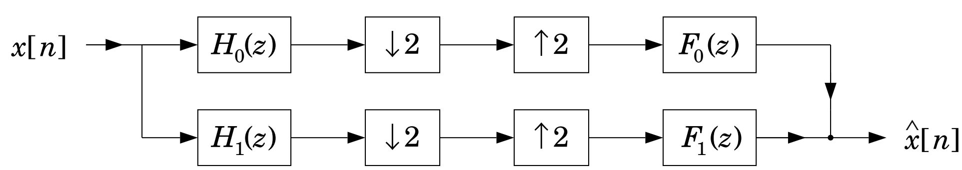

Figure 4.1–1

Problem 4.2

Consider the QMF filter bank shown in Fig. 4.1–1 where $H_1(z) = H_0(-z)$. The filter $H_0(z)$ can be FIR or IIR. However, assume that it is causal and stable.

(a) Assume that the polyphase components $E_0(z)$ and $E_1(z)$ of $H_0(z)$ have all their zeros outside the unit circle. Find a set of stable synthesis filters so that aliasing and amplitude distortion are both eliminated.

(b) Repeat part (a) in the case that $E_0(z)$ and $E_1(z)$ have some zeros inside and the rest of their zeros outside the unit circle (*i.e.*, no zeros on the unit circle).

Problem 4.3

Consider the modified QMF bank shown in Fig. 4.3–1.

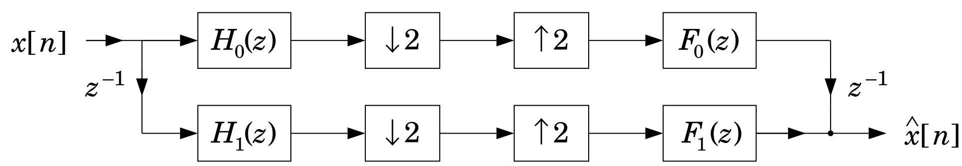

Figure 4.3–1

(a) Express $\hat{X}(z)$ in terms of $X(z)$, the $H_i(z)$'s, and the $F_i(z)$'s.

In parts (b), (c), and (d) assume that the analysis filters satisfy $H_1(z) = H_0(-z)$.

(b) Show that the choice $F_0(z) = H_0(z)$ and $F_1(z) = H_1(z)$ results in an alias-free system. For this choice of synthesis filters express the distortion $T(z)$ in terms of $H_0(z)$.

(c) Let $H_0(z)$ be a real-coefficient linear-phase FIR lowpass filter of order N. Simplify $T(z)$ and show that there is no phase distortion. Also show that N has to be even to ensure that $T(e^{j\pi/2}) \neq 0$.

(d) For the system in part (c) with N even, what is the number of MPUs required to implement the analysis bank?
Hint: Try to exploit as many of the following facts as possible:

(i) the relationship $H_1(z) = H_0(-z)$;

(ii) the linear-phase property;

(iii) the presence of decimators in the system.

Problem 4.4

Consider the analysis/synthesis bank cascade in Fig. 4.4–1.

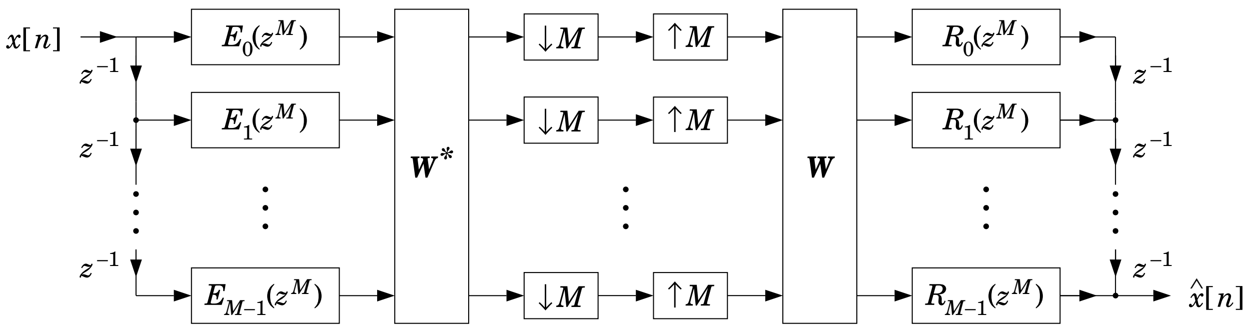

Figure 4.4–1

Assume that all the analysis bank polyphase components $E_i(z)$ are causal FIR filters. Let N denote the maximum filter order among the $E_i(z)$'s (*i.e.*, $e_i[n] = 0$ for all $n > N$ and all i). Also assume that the $E_i(z)$'s have no zeros on the unit circle, but can have zeros inside and outside the unit circle.

(a) Determine a choice of causal FIR synthesis bank polyphase components $R_i(z)$ so that the overall system is free from aliasing.

(b) Determine a choice of causal FIR synthesis bank polyphase components $R_i(z)$ so that the overall system is free from aliasing and phase distortion.

(c) Determine a choice of stable and right-sided synthesis bank polyphase components $R_i(z)$ so that the overall system is free from aliasing and amplitude distortion.

Problem 4.5

Consider the three-channel QMF bank shown in Fig. 4.5–1, in the following three cases where the analysis filters are given by

(i) $H_0(z) = 1,\quad H_1(z) = 2 + z^{-1},\quad H_2(z) = 3 + 2\,z^{-1} + z^{-2}.$

(ii) $H_0(z) = 1,\quad H_1(z) = 2 + z^{-1} + z^{-5},\quad H_2(z) = 3 + 2\,z^{-1} + z^{-2}.$

(iii) $H_0(z) = 1,\quad H_1(z) = 2 + z^{-1} + z^{-5},\quad H_2(z) = 3 + z^{-1} + z^{-2}.$

In each case determine whether it is possible to obtain a perfect reconstruction QMF bank with a set of FIR synthesis filters. If not, determine a set of stable IIR filters for perfect reconstruction.

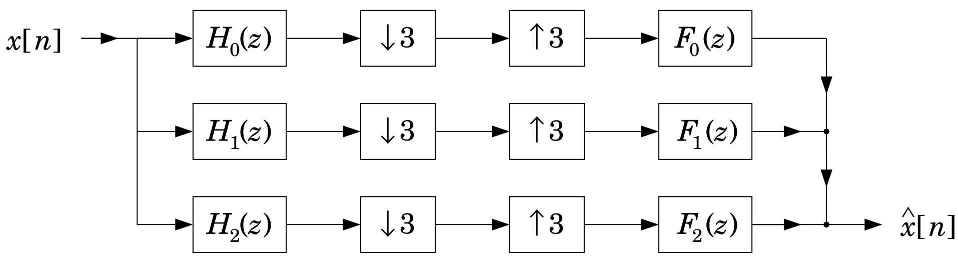

Figure 4.5–1

Problem 4.6 (optional)

Prove that the cascade of the analysis and synthesis banks shown in Fig. 4.6–1 is a perfect reconstruction system provided that M and N are relatively prime. The matrices $\mathbf{W}$ and $\mathbf{W}^*$ are the $M \times M$ DFT and IDFT matrices, respectively.

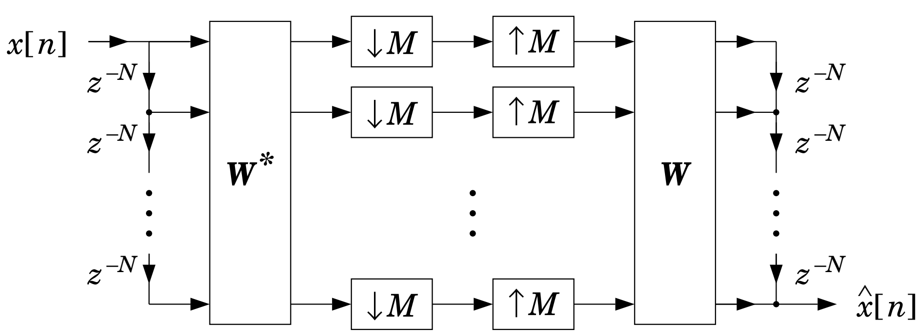

Figure 4.6–1

Problem 4.7

Tree structures can be designed to obtain nonuniform QMF banks, that is, QMF banks for which the decimation factor is not the same for all channels. Consider the filter bank shown in Fig. 4.7–2 (on page 7). The overall system can be described as in Fig. 4.7–1, where $G_k(z)$ and $P_k(z)$ are given in terms of the filters $H_k(z)$ and $F_k(z)$.

(a) Determine the filters $G_k(z)$ and $P_k(z)$ in terms of the filters $H_k(z)$ and $F_k(z)$. Show that, in the case of interest, where $H_0(z)$ and $H_1(z)$ are lowpass and highpass filters, respectively, each with bandwidth ss, the filters $G_k(z)$ and $P_k(z)$ have unequal bandwidths.

(b) Suppose that $F_k(z)$ and $H_k(z)$ in Fig. 4.7–2 are selected so that the 2-channel QMF bank (e.g., the one shown in Fig. 4.1–1) is a perfect reconstruction system, with distortion function $T(z) = 1$. Show that the system in Fig. 4.7–2 is also a perfect reconstruction system.

(c) Suppose that $F_k(z)$ and $H_k(z)$ in Fig. 4.7–2 are selected so that the 2-channel QMF bank (e.g., the one shown in Fig.4.1–1) is free from aliasing. Does this imply that the system in Fig. 4.7–2 is free from aliasing? If yes, justify your reasoning. If no, modify the structure so that it is free from aliasing.

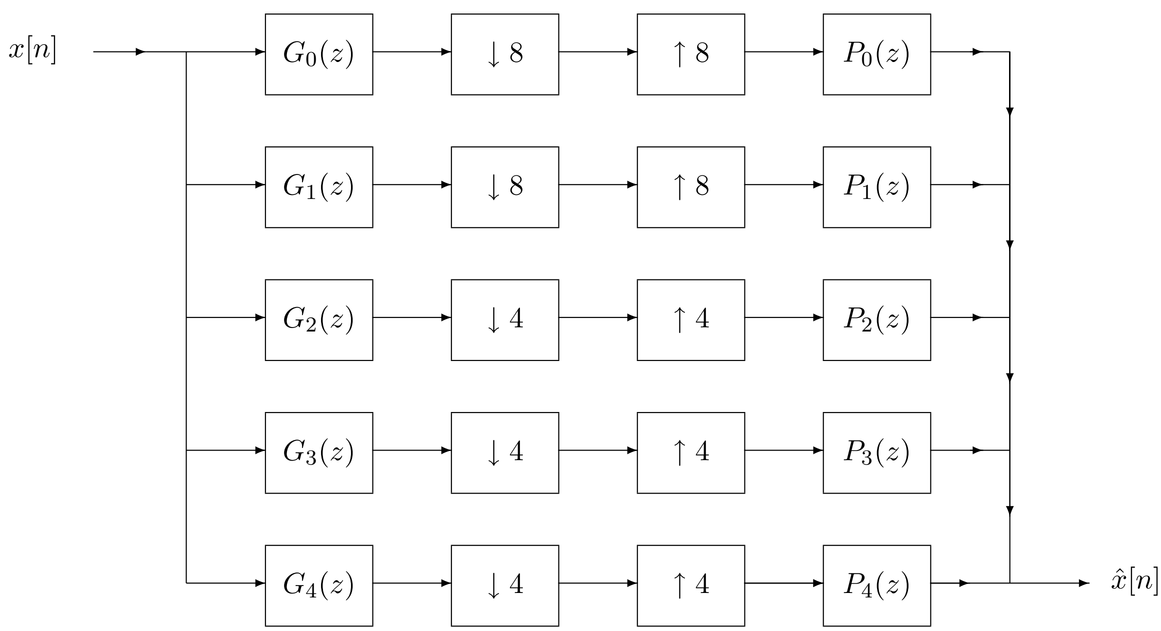

Figure 4.7–1 Nonuniformly Decimated Filter Bank

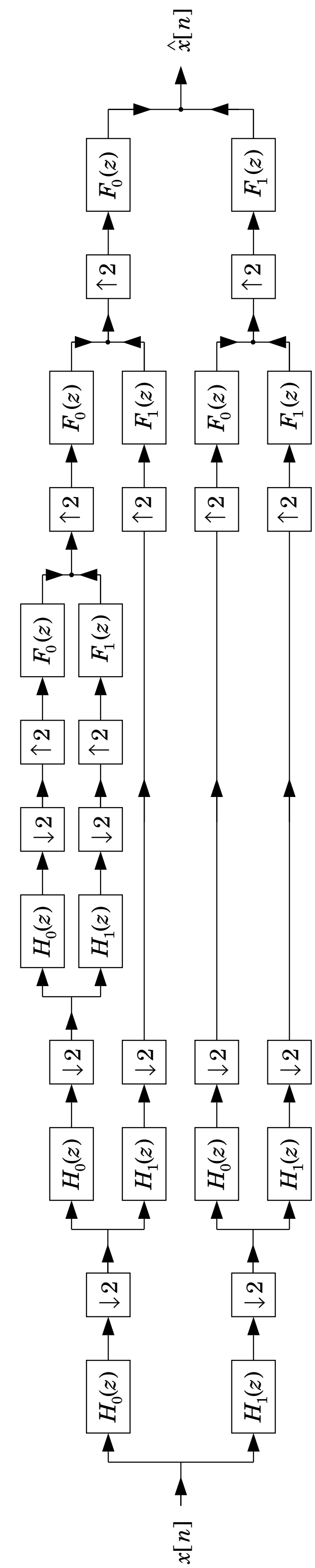

Figure 4.7–2