University of Maryland at College Park
Department of Electrical and Computer Engineering

ENEE 624 Advanced Digital Signal Processing

Problem Set 3

Spring 2003

Issued: Monday, February 24, 2003 **Due:** Wednesday, March 5, 2003

Reading Assignment: Lectures 6–8 (*Vaidyanathan*, Chapter 4, Sections 4.3–4.5.2, 4.6.2A, and 4.6.2C).

Exercise 3.A (do not turn in)

Determine the polyphase components in the Type 1 representation with $M = 2$ for the following filters:

(a) a filter with impulse response $h[n] = 3^{-n}\,(u[n] - u[n-8])$;

(b) a filter with impulse response $h[n] = 2^{-n}\,u[n] - 3^{-n}\,u[n-3]$.

Problem 3.1

Consider the two systems shown in Fig. 3.1–1 where

$$H(z) = \sum_{n=0}^{N} h[n]z^{-n}\,, \text{ and } G(z) = \sum_{n=0}^{N} h[N-n]z^{-n}\,.$$

Draw implementation structures for each of the two systems in Fig. 3.1–1 that use only $N+1$ multiplies per unit time.

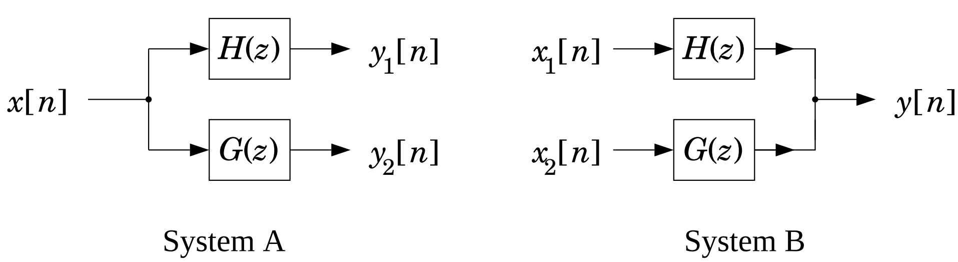

Figure 3.1–1

Problem 3.2 Consider the uniform DFT analysis bank in Fig. 3.2–1 with $M = 4$. Assume

$$E_0(z) = 1 + z^{-1}\,, \quad E_1(z) = 1 + 2\,z^{-1}\,, \quad E_2(z) = 2 + 4\,z^{-2}\,, \quad E_3(z) = 0.5 + z^{-1}\,.$$

Determine $H_k(z)$ for $0 \le k \le 3$.

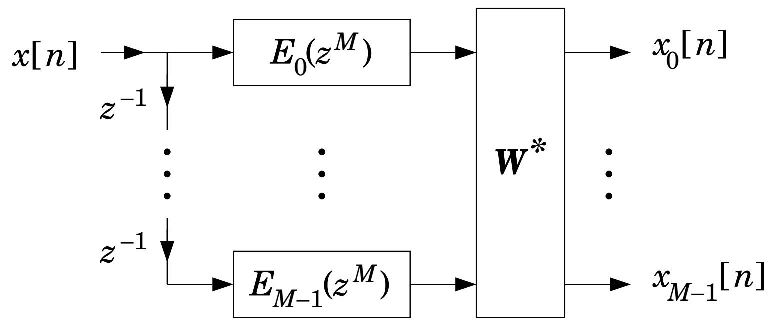

Figure 3.2–1

Problem 3.3

Prove that the DFT synthesis bank in Fig. 3.3–1 (which takes as inputs the outputs of the DFT analysis M-filter bank and results in perfect reconstruction of the input) has synthesis filters that are given by

$$F_k(z) = W^{-k}\, H_0\left(z\, W^k\right) \,,$$

where $W = e^{-j2\pi/M}$, and where $H_0(z)$ denotes the prototype filter of the DFT analysis bank. Note that $\mathbf{W}$ in Fig. 3.3–1 denotes the $M \times M$ DFT matrix.

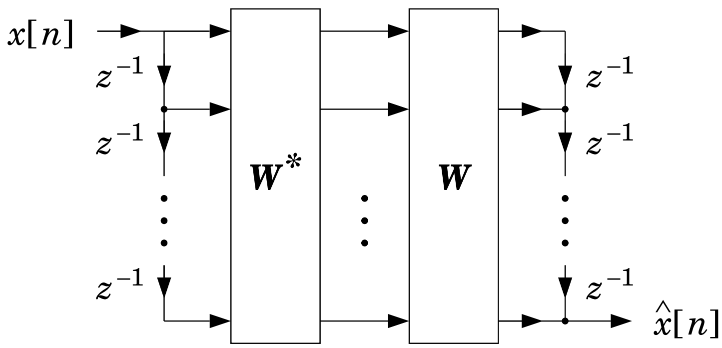

Figure 3.3–1

Problem 3.4

Consider the filter bank in Fig. 3.4–1, where $\mathbf{W}$ is the 3×3 DFT matrix. This is equivalent to a three-filter synthesis bank with synthesis filters $F_0(z)$, $F_1(z)$, and $F_2(z)$.

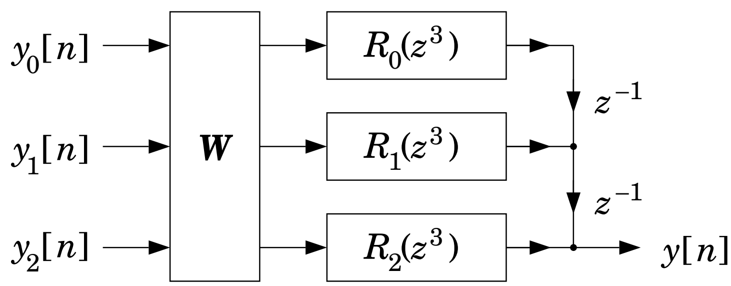

Figure 3.4–1

(a) Determine the three synthesis filters when

$$R_0(z) = 1 - z^{-1}, \quad R_1(z) = 3 + 2\,z^{-1}, \quad R_2(z) = 4 - 2\,z^{-2} .$$

(b) Let the magnitude response of $F_1(z)$ be as shown in Fig. 3.4–2. Plot the magnitude response of $F_0(z)$ and $F_2(z)$.

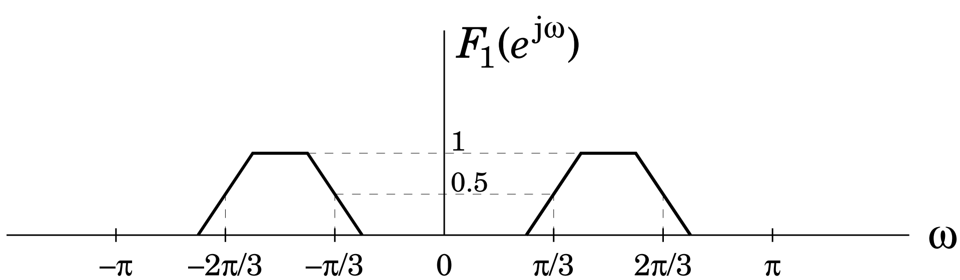

Figure 3.4–2

Problem 3.5

Consider the analysis/synthesis bank in Fig. 3.5–1.

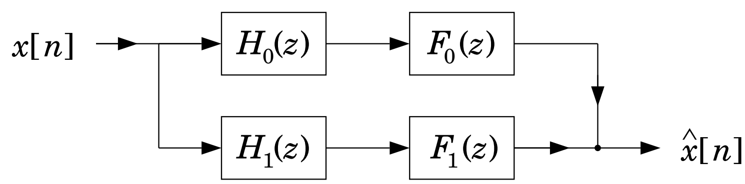

Figure 3.5–1

(a) Let the analysis filters be

$$H_0(z) = 1 + 3\,z^{-1} + 0.5\,z^{-2} + z^{-3}\,,$$

and $H_1(z) = H_0(-z)$. Find causal IIR filters $F_0(z)$ and $F_1(z)$ such that $\hat{x}[n]$ agrees with $x[n]$ except for a possible delay and a (nonzero) scale factor.

(b) Let the analysis filters be

$$H_0(z) = 1 + 2\,z^{-1} + 3\,z^{-2} + 2\,z^{-3} + z^{-4}\,,$$

and $H_1(z) = H_0(-z)$. Find causal FIR filters $F_0(z)$ and $F_1(z)$ such that $\hat{x}[n]$ agrees with $x[n]$ except for a possible delay and a (nonzero) scale factor.

Problem 3.6

In this problem $E_0(z)$ and $E_1(z)$ denote the polyphase components of $H(z)$ for $M = 2$.

(a) Construct a stable allpass filter with a rational transfer function $H(z)$ such that the zeroth polyphase component $E_0(z)$ is also a stable allpass filter.

(b) Determine the relationship that the phase responses of $E_0(z)$ and $E_1(z)$ must satisfy, in the case that $H(z)$, $E_0(z)$, and $E_1(z)$ are all stable allpass filters.

(c) Can you construct a stable allpass filter with a rational transfer function $H(z)$ such that $E_0(z)$ and $E_1(z)$ are both stable allpass filters with rational transfer functions?

Hint: Let $\phi(\omega)$ denote the phase response of a first-order allpass filter (clearly, with a rational transfer function). Then:

(i) $\phi(\omega)$ is either monotonically decreasing or monotonically increasing in $[-\pi, \pi]$.

(ii) The range spanned by $\phi(\omega)$ as ω is varied from $-\pi$ to π equals either $2\,\pi$ or $-2\,\pi$.

Problem 3.7

In this problem we examine the interpolated FIR filter (IFIR) design method shown in Fig. 3.7–1 in the context of narrowband lowpass filter design.

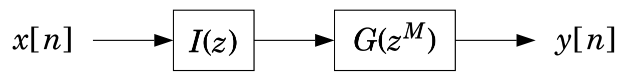

Figure 3.7–1

(a) Let $H(z)$ denote the cascade of the two filters $I(z)$ and $G(z^M)$ resulting from the IFIR design method. Let ω_p, ω_s, δ_p, and δ_s denote the desired passband edge, stopband edge, passband ripple, and stopband ripple, respectively, for the narrowband lowpass filter $H(z)$. Let ω_{p_G}, ω_{s_G}, δ_{p_G}, and δ_{s_G} denote the associate quantities for $G(z)$, and ω_{p_I}, ω_{s_I}, δ_{p_I}, δ_{s_I} denote the associate quantities for $I(z)$. Assume that $G(z)$ and $I(z)$ are designed via the FIR equiripple design method. Assuming also that $M\omega_s < \pi$, determine sets of specifications for $G(z)$ and $I(z)$ (*i.e.*, how to select ω_{p_G}, ω_{s_G}, δ_{p_G}, δ_{s_G}, ω_{p_I}, ω_{s_I}, δ_{p_I}, δ_{s_I} based on the specifications for $H(z)$) so that the resulting $H(z)$ meets the desired specifications.

(b) We wish to design a linear-phase FIR filter with the following specifications:

$$\omega_p = 0.0525\pi\,, \quad \omega_s = 0.0725\pi\,, \quad \delta_p = 0.01\,, \quad \delta_s = 0.001\ .$$

Use the *remez* algorithm in Matlab to design the shortest-length linear-phase FIR filter that achieves these specifications.

(c) Design a linear-phase FIR filter that achieves the specifications in part (b) by employing the IFIR method in Matlab in the cases $M = 2$, 4, 6, and 8. Determine the minimum number of multiplies that are required in each case to achieve the desired specifications and comment on your results.

(d) The FIR equiripple design algorithm designs a linear-phase FIR filter that is optimal in achieving the desired specifications. On the other hand, the IFIR design method outperforms the equiripple design method when designing narrowband lowpass filters. Explain what exactly is meant by the above statements and why they are not contradictory.