University of Maryland at College Park
Department of Electrical and Computer Engineering

ENEE 624 Advanced Digital Signal Processing

Problem Set 2
Spring 2003

Issued: Wednesday, February 5, 2003 **Due:** Monday, February 17, 2003

Reading Assignment: Lectures 3–5 (*Vaidyanathan*, Chapter 4, Sections 4.0–4.2).

Exercise 2.A (do not turn in)

Show that the systems corresponding to decimation by M and upsampling by L, where L, M are integers $L, M \geq 2$, are linear time-varying systems.

Exercise 2.B (do not turn in)

Consider the two systems in Fig. 2.B–1.

Figure 2.B–1

(a) Describe $y_1[n]$ and $y_2[n]$ in terms of $x[n]$, M, and L.

(b) Use the relationships that you derived in part (a) to show that, if M and L are relatively prime, then $y_1[n] = y_2[n]$.

Exercise 2.C (do not turn in)

Consider the two sets of M numbers given by W^k for $0 \leq k \leq M-1$, and W^{kL} for $0 \leq k \leq M-1$, where $W = e^{-j2\pi/M}$. Show that these sets are identical if L and M are relatively prime.

Problem 2.1

Determine whether each of the systems in Fig. 2.1–1 is time invariant, periodically time varying, or time varying. In each case where the system is time invariant, determine the impulse response or the frequency response of the system. Assume that in all systems in the figure $L \geq 2$.

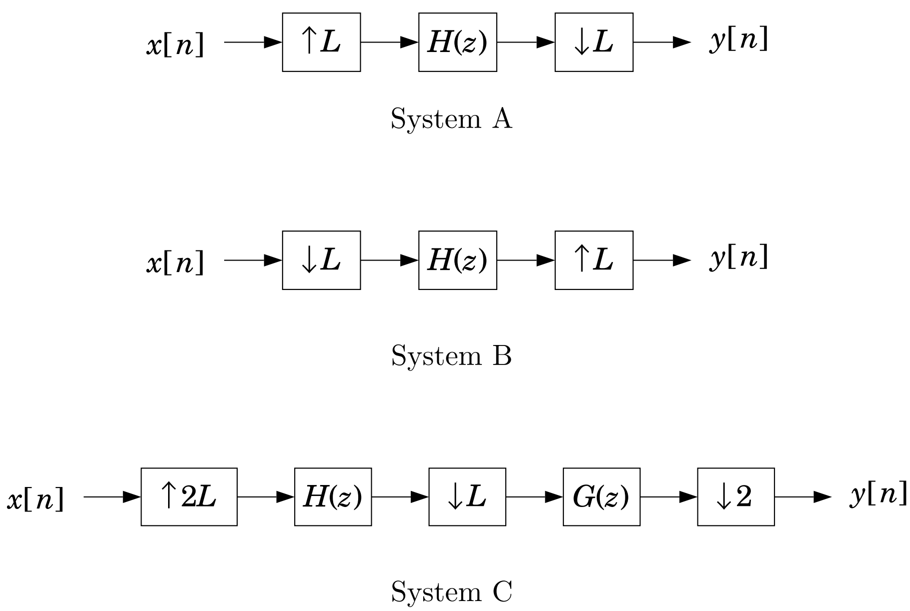

Figure 2.1–1

Problem 2.2

Show that the two systems in Fig. 2.2–1 (where k is some integer) are equivalent (that is, $y_0[n] = y_1[n]$) when

$$h_k[n] = h_0[n]\,\cos(2\pi nk/L)\ .$$

This result implies that, following an upsample by L operation, filtering followed by cosine modulation (with a cosine whose frequency is an integer multiple of $2\pi/L$) has the same effect as filtering with the corresponding cosine modulated impulse response.

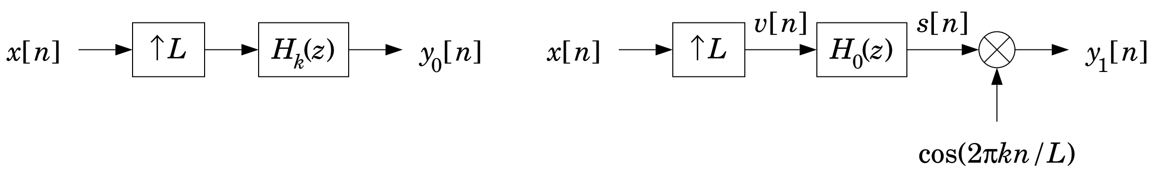

Figure 2.2–1

Sketch $V(e^{j\omega})$ $S(e^{j\omega})$ and $Y_0(e^{j\omega})$ when $L = 3$ and $k = 1$, in the case that $H_0(e^{j\omega})$ is an ideal lowpass filter with cut-off frequency $\pi/3$ and passband amplitude 1, and $X(e^{j\omega})$ is as shown in Fig. 2.2–2.

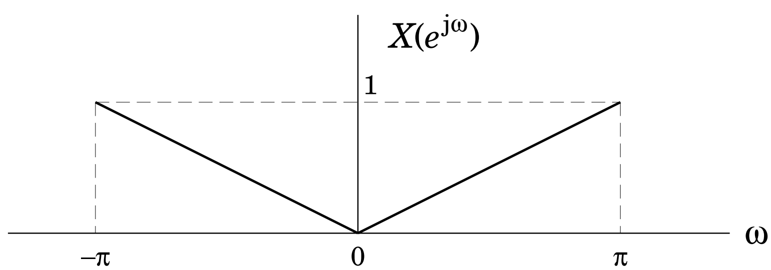

Figure 2.2–2

Problem 2.3

Show that the two systems shown in Fig. 2.3–1, where

$$h_k[n] = h_0[n]\,\cos(2\pi kn/L)\,,$$

are not equivalent, that is, in general, $y_0[n]$ and $y_1[n]$ are not equal.

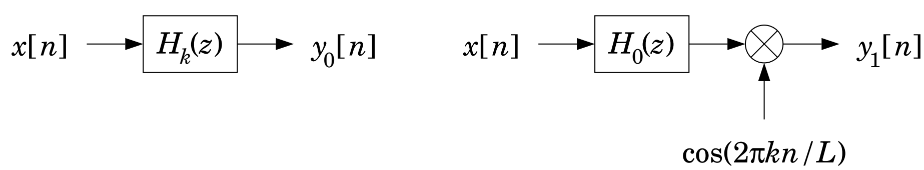

Figure 2.3–1

Problem 2.4

Let $x[n]$ be a periodic signal with fundamental period $N \geq 2$, *i.e.*, N is the smallest positive integer such that $x[n-N] = x[n]$ for all n. Let $y[n]$ be the M-fold decimation of $x[n]$, that is, $y[n] = x[Mn]$. Show that $y[n]$ is periodic, *i.e.*, there exists an integer L such that $y[n-L] = y[n]$ for all n. Assuming no further knowledge about the input what is the fundamental period of $y[n]$ in terms of N and M?

Problem 2.5

Consider a sequence $x[n]$ with $X(e^{j\omega})$ as shown in Fig. 2.5–1.

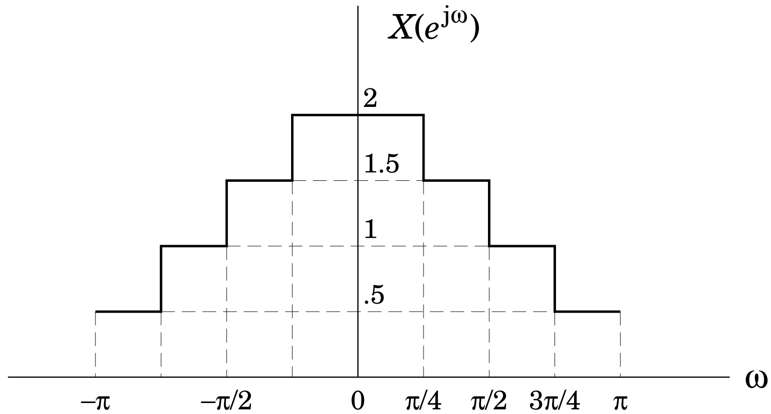

Figure 2.5–1

Suppose we generate $s[n]$ and $y[n]$ from $x[n]$ as in Fig. 2.5–2,

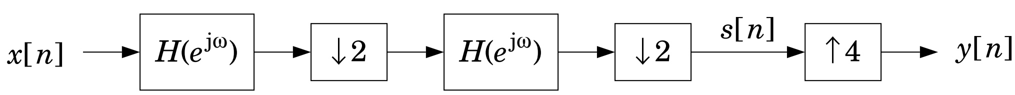

Figure 2.5–2

where $H(e^{j\omega})$ is an ideal low pass filter

$$H(e^{j\omega}) = \begin{cases} 1 & \text{for } |\omega| < \pi/2 \\ 0 & \text{for } \pi/2 \leq |\omega| \leq \pi \end{cases} .$$

Plot the quantities $S(e^{j\omega})$ and $Y(e^{j\omega})$.

Problem 2.6

Simplify the following systems as much as possible. In each case clearly state your reasoning.

(a) The system in Fig. 2.6–1, where M, L are integers, and $M, L \geq 2$.

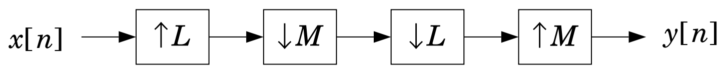

Figure 2.6–1

(b) The system in Fig. 2.6–2.

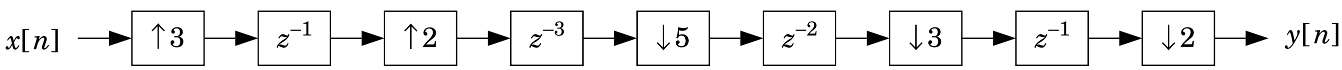

Figure 2.6–2

(c) The system in Fig. 2.6–3.

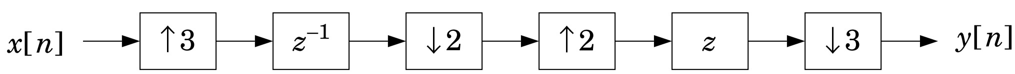

Figure 2.6–3

Problem 2.7

Consider the following statement: "If $g[n]$ is an allpass filter and $g[n] = h[2n]$, where $h[n]$ is also an allpass filter, then $g[n]$ must be of the form $g[n] = c\,\delta[n-k]$ for some complex scalar c and some integer k."

(a) Show that the statement is false by first determining the equation that the frequency responses of the two filters must satisfy and then constructing a counterexample by selecting the phase of the frequency response of $h[n]$ appropriately.

(b) Assume the filter $h[n]$ has a rational transfer function $H(z)$. Is the statement true in this case? Justify your reasoning.